Economic Insights – Trends and Challenges	Vol.IV(LXVII) No. 2/2015	117 – 124

Modeling Sovereign Risk Interaction in Emerging Europe

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Abstract

In this study we implement a flexible model to assess both the strength and the pattern of sovereign risk dependence between several European countries with emerging economies. We employ a market risk approach to analyzing sovereign risk and provide a synoptic interpretation of the results facilitating a better understanding of the interconnectedness of sovereign risk. We conclude that sovereign risk is not a country specific type of risk but rather a reflection of both internal factors and macroeconomic forces external to the country.

Keywords: *copula; garch; hac; sovereign risk*

JEL Classification: *C22; C46; H63*

Introduction

The purpose in this study is to implement a flexible model to assess both the strength and the pattern of sovereign risk dependence between several European countries. We aim at giving a synoptic interpretation to sovereign risk, in the sense that we would like to see whether the risks associated to these countries move in tandem or they can be analyzed independently. We pursue a novel implementation by using an extensive data set of credit default swap (CDS) contracts on the debt issued by 11 European emerging countries. This market risk approach to analyzing sovereign risk has at least two advantages when compared to more traditional macroeconomic approaches: i) as CDS spreads reflect the market perspective on the credit risk of a particular country we believe that these indicators are tightly linked to the general health of the country’s finances. In addition, using market data improves accuracy because it reacts promptly to changes in the macroeconomic environment; and ii) CDS contracts are more liquid than the corresponding sovereign bond market allowing for a cleaner extraction of the credit risk premium from the CDS spread.

Given the rapid expansion of sovereign debt markets, understanding the nature of sovereign risk is of increasing importance because it directly affects the cost and flow of capital across countries. Furthermore, understanding sovereign risk and its interconnectedness improves the ability of market participants to assess the implications of their investments, especially with respect to diversification of debt portfolios. Existing literature focuses more on the incentives and ability of sovereign debtors to repay their debt and, despite its importance, there is relatively little research on the sources of commonality and dependence in sovereign risk. Of particular relevance are the studies of Remolona et al. (2008) who map agency ratings to default losses and then decompose sovereign credit spreads into credit risk and premium components for a panel of 24 countries. Pan et at (2008) use an affine sovereign credit model to demonstrate how common factors could induce significant correlation among credit spreads. Our approach is different in that we seek to describe the interconnectedness of sovereign risk rather than predict individual country default. We concur that describing an economy by an index might overlook some information but at the same time we argue that it adds simplicity and produces more realistic results because it reduces the model dependency on extensive sets of assumptions. Our implementation is a combination between the breadth provided by high dimensionality and the generality of using aggregated indicators.

The literature on copula modeling is growing by the day. For a thorough introduction to copulas we refer to Joe (1997) and Nelsen (2006), the two key text books on dependence modeling from a statistical perspective. McNeil et al. (2005) provides a sound implementation of copula models in the context of quantitative risk management while Embrechts et al. (2002) introduces the static representation of dependence via copulas. Patton (2006) lays down the foundation for multivariate financial time series applications of copulas and complements his research with a comprehensive empirical study in Patton (2012). Choros et al. (2010) presents the parametric and semi-parametric estimation methods for copulas on time series data. Patton (2009) gives an overall survey of copula applications to time series

The paper proceeds as follows: section 2 presents the modeling framework, section 3 describes the implementation and section 4 concludes.

Modeling Framework

From a methodological perspective, our goal is to disentangle the idiosyncratic components from the common factors driving the sovereign risk. We employ a copula-GARCH approach and proceed by dividing the study in two phases. First, we filter the univariate series to extract all temporal dependence. The resulting cross sectional panel of standardized residuals reflects only the pure joint dependence. Second, we fit a multidimensional hierarchical Archimedean copula (HAC) to describe the pattern of association. One advantage of the copula-GARCH approach is the possibility to specify and estimate the model in stages. The marginal distributions are specified by an ARMA-GARCH model for each univariate time series and then a copula (in this case a HAC) is estimated on the probability integral transforms of the standardized residuals. The result is a valid multi-dimensional joint distribution that is easier to estimate and interpret.

The seminal finding in the copula modeling literature is due to Sklar (1959) who provides the formal approach to separate a joint distribution into independent margins and a copula. Accordingly, for every p -dimensional distribution F with corresponding margins F_i there exists a copula C such that:

$$F(y_{i}1, \ldots, y_{i}p) = C(F_{i}1\,(y_{i}1), \ldots, F_{i}p\,(y_{i}p)) \tag{1}$$

which is unique if all margins are continuous. Conversely:

$$C\left(u_1, \ldots, u_p\right) = F\left(F_1^{-1}(u_1), \ldots, F_p^{-1}(u_p)\right) \tag{2}$$

where $u_i = F_i(y_i), i = 1, \ldots, p$. If F_i is continuous then the probability integral transformation $U_i = F_i(y_i)$ is unique and $Unif\,(0, 1)$ distributed regardless of the original distribution of F_i . If F is p -times differentiable then the joint density is given by:

$$f(y) = \frac{\partial^p}{\partial y_1 \partial y_2, \dots, \partial y_p} F(y)$$
$$= \prod_{i=1}^{p} f_i(y_i) \frac{\partial^p}{\partial u_1 \partial u_2, \dots, \partial u_p} C(F_1(y_1), \dots, F_p(y_p)) \quad (3)$$

The copula-GARCH class of models assumes certain parameters are time varying in an auto-regressive manner and their distributions are conditional on past information. In the context of this analysis we are interested in modeling the cross sectional dependence between time series data and therefore we employ an adapted version of Sklar's theorem introduced by Patton (2006). The multivariate distribution $\mathbf{F}$ of a vector $\mathbf{Y_t}$, conditional on the information set available at time $\mathbf{t-1}$ given by $\mathcal{F}_{t-1} = \{Y_l : l \leq t-1\}$, is decomposed into its conditional margins $\mathbf{F_i}$ and the corresponding conditional copula in the following way:

$$F(y|\mathcal{F}_{t-1}) = C\{F_1(y_1|\mathcal{F}_{t-1}), \dots, F_p(y_p|\mathcal{F}_{t-1})|\mathcal{F}_{t-1}\}$$
$$\text{with } Y_it\ |\mathcal{F}_i(t-1) \sim F_i i\ (\cdot\ |\mathcal{F}_i(t-1)), i = 1, \dots, p \quad (4)$$

Fitting a copula on the unconditional probability integral transform will result in an unconditional copula model for the dependence. In a time series context however, it is necessary to condition on the available past information which first requires the specification of the margins and then the copula that joins the series cross-sectionally. If we define the probability integral transform $U_{it} = F_i(Y_{it}|\mathcal{F}_{t-1})$ then the conditional copula of $Y_it\ |\mathcal{F}_i(t-1)$ is given by $U_it\ |\mathcal{F}_i(t-1) \sim C(\cdot\ |\mathcal{F}_i(t-1))$. It is important to note that both the margins and the copula have to be conditional on the same data set.

We use the standard ARMA-GARCH approach to model the univariate distributions by specifying the following general model for each univariate series:

$$Y_{it} = \mu_i(Y_{t-1}) + \sigma_i(Y_{t-1})\epsilon_{it} \quad (5)$$

where $\epsilon_{it} \sim F_i(0,1), \forall\ t.$ The conditional copula is fitted on the conditional distribution of the probability integral transform of the standardized residuals constructed as:

$$\hat{\epsilon}_{it} = \frac{Y_{it} - \mu_i(Y_{t-1})}{\sigma_i(Y_{t-1})}, i = 1, 2 \dots, p \quad (6)$$

The parametric form of $\mathbf{F_i}$ has to be able to accommodate thicker than normal tails and possibly an asymmetric shape. For this analysis, we tested both the normal distribution and the Student-t for its ability to control the thickness of the tails via the degrees of freedom parameter.

Archimedean copulas are related to the Laplace transforms of univariate distribution functions. According to Joe (1997)if we denote by $\mathbb{L}$ the class of Laplace transforms that consist of strictly decreasing differentiable functions than the function $C: [0,1]^d \to [0,1]$ defined as:

$$C(u_1, \dots, u_n; \theta) = \phi\{\phi^{-1}(u) + \dots + \phi^{-1}(u_d)\}, u_1, \dots, u_d \in [0,1] \quad (7)$$

is a d-dimensional exchangeable Archimedean copula where $\phi \in \mathbb{L}$ is called the generator function and θ is the copula parameter. Archimedean copulas provide an elegant solution to accommodate tail dependence in non-elliptical distributions. However, fitting a fully nested structure to a large data set is unfeasible. This disadvantage comes from the fact that the multivariate dependence structure typically depends on a single parameter of the generator function. Furthermore, the resulting distribution is exchangeable which means the dependence is symmetric with respect to the permutation of the variables. HACs alleviate these shortcomings by providing an efficient way to recursively define the dependence structure for large dimensional data sets. Using the same notation as in (7), a fully nested HAC connecting $d-1$ nesting levels is defined recursively by the following relation:

$$\begin{aligned} C(u_1,\dots,u_d) &= \phi_{d-1}\{\phi_{d-1}^{-1}\circ\phi_{d-1}\{\cdots[\phi_2^{-1}\circ\phi_1\{\phi_1^{-1}(u_1)+\phi_1^{-1}(u_2)\}+\phi_2^{-1}(u_3)]+\cdots\phi_{d-2}^{-1}(u_{d-1})\}+\phi_{d-1}^{-1}(u_d)\} \\ &= \phi_{d-1}\{\phi_{d-1}^{-1}\circ C(\phi_1,\dots,\phi_{d-1})(u_1,\dots,u_{d-1})+\phi_{d-1}^{-1}(u_d)\} \\ &= C_{d-1}\{C_{d-2}(u_1,\dots,u_{d-1}),u_d\} \end{aligned} \quad (8)$$

According to Okhrin et al. (2013) such a structure is determined recursively starting at the lowest level with a copula ϕ_1 forming a variable $z_1=\phi_1\{\phi_1^{-1}(u_1)+\phi_1^{-1}(u_2)\}$. At the second level another copula is used to capture de dependence between z_1 and u_3 and so on. The generators ϕ_i may come from the same family and differ only in parameter or may come from different generator families. Okhrin et al. (2013) propose an efficient method to determine the optimal structure. The estimation procedure relies on a recursive multi-stage maximum likelihood method which determines the parameters at each level and the structure simultaneously (the structure itself is in fact a parameter to estimate).

Model Implementation and Results

We apply the methodological framework described above to an extensive data set of CDS contracts on the sovereign debt of 11 European emerging economies. A key reason for our choice of CDS spreads is their sensitivity to changes in market perception of the probability of default. Conrad et al. (2011) conducts an illustrative empirical study on implied probability of defaults derived from CDS spreads. Briefly, a CDS is a contract whereby the seller provides insurance to the buyer against the losses resulted from a default by the reference entity within the specified horizon. Investors can express their bullish or bearish sentiments related to the credit risk as an asset class which makes the CDS contracts very efficient as forward looking indicators. Given their liquidity and trading characteristics we believe these contracts are representative indicators of credit risk for their respective sovereign bonds. When analyzed together they reveal the interdependencies between the credit risks of each country and therefore create a good reflection of systemic risk at emerging European market level.

The CDS spread is quoted in basis points for different maturities across the credit curve. Our data set consists of the daily 5 year maturity, CDS spreads of 11 countries spanning a period of roughly 5 years. Figure 1 depicts the time evolution of the CDS spreads for each country in the data set. Countries are identified by their ISO country code. The levels of our CDS data are indicative of autoregressive processes and similar to other market data series they are unlikely to follow a random walk (they are also bounded below). We model in log-differences to avoid treating the series as near unit root processes. All data was retrieved from the Bloomberg Database.

Using the methodology of Carr (2011), it can be shown that under some simplifying assumptions (such as constant risk free rate and constant hazard rate) the CDS spread can be expressed as:

$$S_{it} = 100^2 P_{it}^{Q} L_{it} \quad (9)$$

where P_{it}^{Q} is the market implied probability of default and L_{it} is the loss given default (LGD). The same formula can be rewritten in terms of objective probability of default P_{it}^{P} as:

$$S_{it} = 100^2 P_{it}^{P} \mathcal{M}_{it} L_{it} \quad (10)$$

where $\mathcal{M}_{it}$ is the market price of risk. An increase in a CDS spread can be induced by an increase in LGD, an increase in the market perception of default risk or an increase in the objective probability of default. Any one of these effects is indicative of a worsening financial soundness of the respective sovereign. In our analysis we work with the log-returns of CDS spreads to mitigate their autoregressive persistence under the following transformation:

$$y_{it} = \Delta \log S_{it} = \Delta \log P_{it}^{P} + \Delta \log \mathcal{M}_{it} + \Delta \log L_{it} \quad (11)$$

If we follow the business cycle theory then the market price of risk is constant or evolves slowly. If we further assume, in accordance with the common practice, a constant LGD then the second and the third terms in (11) vanish leaving the changes in CDS spreads to be directly attributed to the changes in the empirical probability of default. Table 1 presents the summary statistics of the CDS spreads both in their level form and log-returns. Of particular importance for our analysis is the high level of kurtosis of the log-return series as this imposes the need for a distribution to capture the thickness in the tails (and possibly skewed).

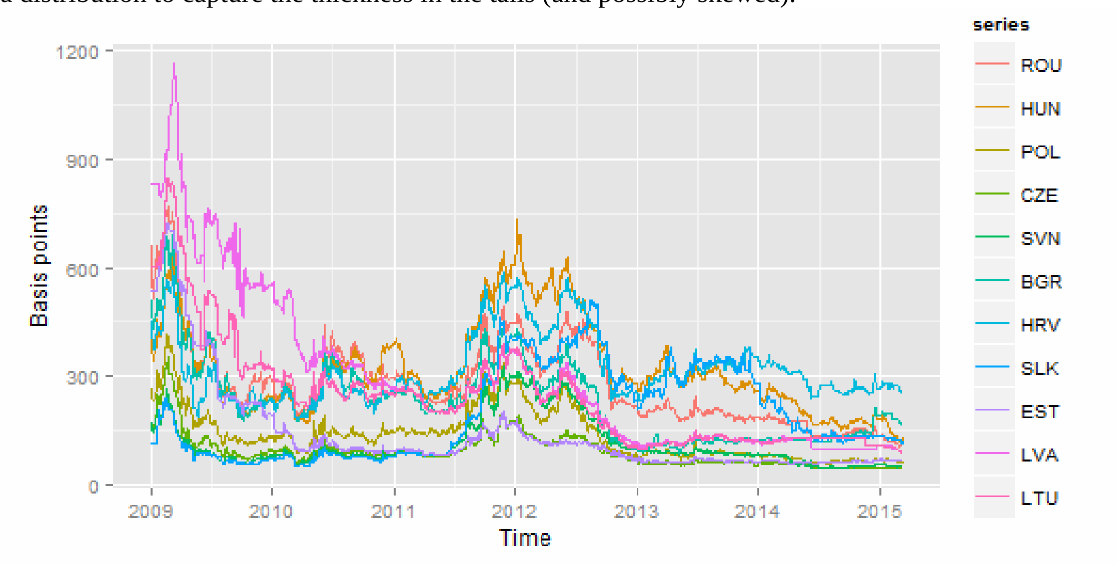

Fig 1. CDS spreads for countries in data set (ISO country codes)

Source: Bloomberg

Table 1. Summary statistics of CDS spreads both in level and log-return forms

	Levels (in basis points)				Log returns			
	1st Qu	**Median**	**Mean**	**3rd Qu**	**Mean**	**Std**	**Skew**	**Kurt**
ROU	189	252	278	344	- 0.0011	0.0286	- 0.3687	7.6017
HUN	238	291	322	381	- 0.0007	0.0292	0.1818	8.8396
POL	82	126	137	166	- 0.0009	0.0365	0.2100	7.0302
CZE	59	82	91	109	- 0.0008	0.0335	- 0.1096	12.7427
SVN	74	85	111	114	- 0.0007	0.0359	- 0.0003	9.5602
BGR	126	215	229	287	- 0.0007	0.0307	0.0317	7.8669
HRV	252	288	318	344	- 0.0004	0.0271	0.3448	10.2133
SVK	81	136	193	302	0.0000	0.0394	1.8607	34.2530
EST	64	93	137	132	- 0.0013	0.0312	- 0.9213	16.5526
LVA	121	245	292	351	- 0.0014	0.0291	- 0.7707	25.9898
LTU	128	227	243	291	- 0.0012	0.0280	- 0.3372	23.3966

Source: Bloomberg, author own calculations

Daily CDS spreads have more autocorrelation (i.e. risk persistence) than is found in other market data. Therefore our conditional mean-variance models need more structure than the commonly used model for daily stock returns. Applying an ARMA-GARCH process to the return series of the two indices removes the temporal correlation but preserves the cross-sectional dependence. As most asset returns, our series are not stationary (at least in the variance) and therefore we followed the standard mean-variance model building approach to make our series temporal independent. The order for the mean equation was determined by comparing the BIC of ARMA models of orders up to 5. An AR(1) process was optimal for most of the series and this decision has also been confirmed by the partial autocorrelation function of the squared log-returns. Then, we used the squared residuals from the mean equation to test for conditional heteroskedasticity. Applying the Ljung-Box test on the first 12 squared log-returns revealed very close to zero p-values. This gives strong indication of rejecting the null and a motivation to introduce a conditional variance equation. To handle the fat tails characteristics we used the Student-t distribution. Taking all the above into consideration we implemented the following form of mean-variance modeling:

$$\begin{gathered} y_{it} = \mu_i + \theta_{i1} y_{i,t-1} + e_{it} \\ e_{it} = \sigma_{it}\epsilon_{it}, \qquad \epsilon_{it} \sim Student - t\ (\nu) \\ \sigma_{it} = \omega_i + \alpha e^2_{i,t-1} + \beta\sigma_{i,t-1} \end{gathered} \tag{12}$$

and the results are presented in Table 2. For ROU and BGR the degrees of freedom parameter came out not significant so we used the normal distribution instead. Similarly, for POL, CZE and SVN the AR (1) parameters were not significant therefore we only used the equation for the variance - GARCH(1,1).The HAC is fitted on the standardized residuals obtained from (12). Taking into account the dependence characteristics for each pair of standardized residuals (judging by the scatterplots) we decided to use the Gumbel generator at each node. HACs have at least two interesting characteristics: first, the structure is recursive which entails that the marginal distribution at each node in the tree is also a HAC. However for ease of interpretation we opted for a fully nested HAC (i.e. binary copula at each node); second, if the same copula, with a single parameter is used at each level then the parameters should increase with the levels. This provides an intuitive interpretation of the copula tree as the dependence at the bottom is stronger than at the top. In addition, we chose to present the results in Figure 2 using the equivalent Kendall's τ as it is easier to interpret than the respective copula parameter (τ represents the rank correlation and is bounded by $[0,1]$). Copula estimation was performed in R using the HAC package by Okhrin et al. (2014).

Table 2. Parameter estimates of the ARMA-GARCH processes

	μ	θ_1	ω	α	β	ν
ROU	-0.0013	0.1441	0.0001	0.1540	0.7331	
	(0.0006)	(0.0309)	(0.0000)	(0.0286)	(0.0420)	
HUN	-0.0011	0.1275	0.0001	0.2334	0.7594	5.1850
	(0.0005)	(0.0252)	(0.0000)	(0.0726)	(0.0661)	(0.3167)
POL	-0.0018		0.0001	0.2576	0.7140	4.4670
	(0.0006)		(0.0000)	(0.0520)	(0.0317)	(0.3593)
CZE	0.0090		0.0000	0.3225	0.4894	3.5680
	(0.0043)		(0.0000)	(0.0747)	(0.0189)	(0.0654)
SVN	-0.0010		0.0002	0.4144	0.5556	3.2352
	(0.0004)		(0.0001)	(0.1848)	(0.0464)	(0.1513)

Table 2 (cont.)

BGR	-0.0007	0.0874	0.0001	0.1311	0.8140	
	(0.0003)	(0.0304)	(0.0000)	(0.0224)	(0.0291)	
HRV	-0.0004	0.1161	0.0001	0.2981	0.6160	3.2850
	(0.0003)	(0.0250)	(0.0001)	(0.1566)	(0.0481)	(0.1338)
SVK	-0.0002	-0.0450	0.0004	0.3181	0.6593	2.1472
	(0.0004)	(0.0224)	(0.0001)	(0.1157)	(0.0551)	(0.0522)
EST	-0.0001	-0.0466	0.0000	0.1874	0.7340	3.1830
	(0.0001)	(0.0218)	(0.0000)	(0.0911)	(0.0388)	(0.0398)
LVA	-0.0012		0.0000	0.2586	0.6530	3.5460
	(0.0003)		(0.0000)	(0.0697)	(0.0196)	(0.0546)
LTU	-0.0001		0.0000	0.1795	0.6878	3.4780
	(0.0002)		(0.0000)	(0.0732)	(0.0247)	(0.0466)

Source: Bloomberg, author own calculations

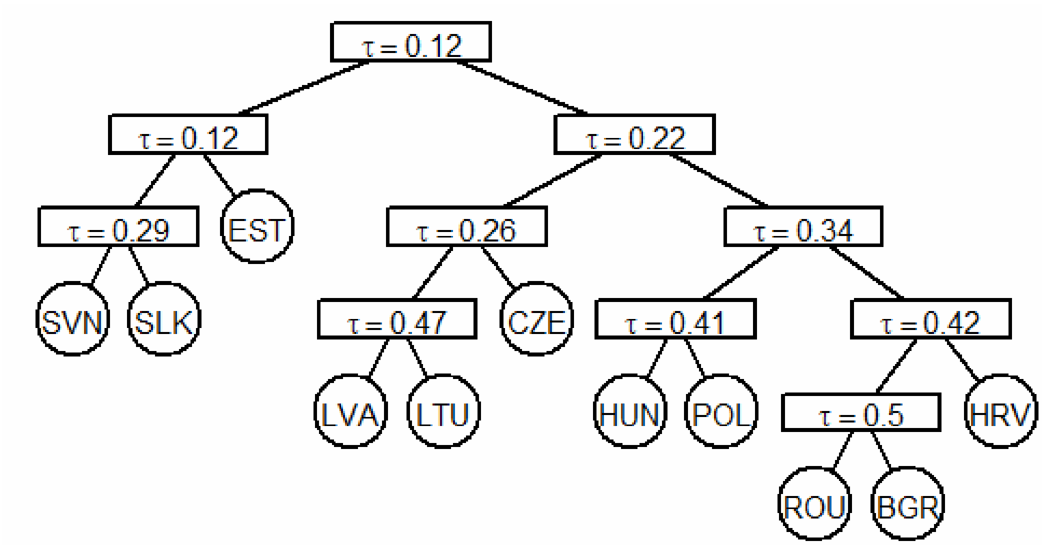

Fig 2. Pattern of sovereign risk dependence

Source: Bloomberg, author own calculations

Conclusions

Understanding the nature of sovereign credit risk is of paramount importance in today's environment of rapidly growing size of the sovereign debt markets. Furthermore, the nature of sovereign credit risk directly affects the ability of financial market participants to diversify the risk of debt portfolios and may play a central role in determining both the cost and flow of capital across countries. We employ a novel market based approach to analyze sovereign credit risk by using an extensive dataset of sovereign credit default swap (CDS) contracts on the tradable debt of 11 eastern European countries. Sovereign CDS contracts function as insurance contracts that allow investors to disentangle the yield attributable to credit risk and purchase protection against the default of a particular country. From a methodological perspective a key advantage of using sovereign CDS data (instead of yields) is that the CDS market is generally more liquid than the corresponding sovereign bond market, translating into more accurate estimates of credit risk. In this study we determined the degree of interconnectivity of sovereign risk and the pattern of association in the credit risk of these countries. Several important conclusions derive from this study: i) first, copula parameters are significant across the tree meaning that sovereign risk is not a country specific type of risk but rather a reflection of both internal factors and macroeconomic forces external to the country; ii) judging by how the countries are grouped it is evident that countries adopting the euro (SVN, SVK, EST) have a different risk profile and a weaker connection to the others; iii) the newest members of the EU – ROU, BGR and HRV are grouped together indicating similar and interconnected sovereign risk. ROU and BGR present the strongest risk dependence, which is in line with the general perception of seeing and referencing these countries together; iv) countries that joined the Eurozone most recently- LVA and LTU are grouped together with CZE, the strongest economy in the group.

Acknowledgment

This work was financially supported through the project "Routes of academic excellence in doctoral and post-doctoral research – REACH" co-financed through the European Social Fund, by Sectoral Operational Programme Human Resources Development 2007-2013, contract no POSDRU/159/1.5/S/137926.

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