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On the Stationarity of Chronological Series

Cornelia Gaber*, Maria Stoica**

* Petroleum-Gas University of Ploieşti, 39 Bd. Bucureşti, Ploieşti, Romania
e-mail: corneliagaber@yahoo.com

** "Nichita Stănescu" High School, 3 Nalbei, Ploieşti, Romania
e-mail: pmagdastoica@yahoo.com

Abstract

The construction of the statistic or econometric models of the chronological series is also made with the help of stochastic processes. This article presents a modeling problem of stationarity as the property of chronological series using the stochastic equation of a chronological series and the stochastic equation of the mobile average.

Key words: *covariant, corelograma , stochastic process*

JEL Classification: *A12*

Introduction

The statistic series presents itself as a multitude of statistic data reflecting yearly, quarterly, monthly results etc. regarding the economic activities which are characterized not by stability but mainly by fluctuations within a specified time period. The analysis of a chronological series can be made by determining some indicators regarding the values variation, by determining the adjusted values using suitable statistic methods and which can not allow obtaining some accurate information regarding the future or anticipated values.

The evolution of a social-economic phenomenon is determined by different factors requiring thus the emphasis of the chronological series nature and of its v_a value as related to the possible result of the phenomenon or uncontrollable complex process F .

Statistically speaking, process F can be considered a positive real aleatory variable V , the variable being defined by a probability field associated to F and the chronological series is defined by $\{v_t / t \in T\}$, $T \subseteq [0,\infty)$, v_t represents the value of an economic variable at time *t*, noted with V_t and defined on the borelian field (Ω, K, P).

The construction of the statistic or econometric series is also made with the help of the stochastic processes. If the space of elementary events, *K* is a borelian body or σ-algebra over Ω and *P* is a completely additional probability or a probabilistic measure on the borlian fields of events $(\Omega, K,)$ and a totally ordered multitude is $(T, \le)$, then we define a stochastic process on (Ω, K, P) and the ordered multitude $(T, \le)$ as the aleatory variables family

$\{V(t,\omega) \mid \omega \in \Omega, t \in T\}$, where $V(t,\omega): T \times \Omega \to R$ has partial functions $V(t,\bullet): \Omega \to R$ as a real onedimensional aleatory variable and $V(\bullet;\omega): T \to R$ which is a real function called achievement or trajectory of the stochastic process and T is called indexes multitude.

The notations for this stochastic process are: $\{V(t,\omega)\}$ sau $(V_t(\omega))_{t\in T}$ or $(V_t(\bullet))_{t\in T}$.

For the fixed $\omega \in \Omega$ the adequate achievement of the stochastic process is noted by $(V_t)_{t\in T}$ where $v_t \overset{noted}{===} V_t(\omega)$.

According to the definition of the stochastic process induced in [3] we can consider the stochastic process an aleatory variables family $(V_t(\bullet))_{t\in T}$, defined on the same probability borelian field (Ω, K, P).

The existence of the stochastic process is presented in [5], the aleatory variables $(V_t(\bullet))_{t\in T}$ are discrete or of continuous type and the schievements of the stochastic process $(V_t(\bullet))_{t\in T}$ can be real continuous functions or not.

If $T = \{t_1, t_2, ..., t_n\}$ is finite then the stochastic process $(V_t(\bullet))_{t\in T}$ presents itself as an aleatory vector $V(\bullet) = (V_{t_j}(\bullet))_{j=\overline{1,n}}$ and probabilistically speaking, it is completely determined by the common repartition function:

$$F_{t_1,t_2,...,t_n}(v_1, v_2, ..., v_n) = P\left(\{\omega | V_{t_1}(\omega) < v_1, ..., | V_{t_j}(\omega) < v_j, .., | V_{t_n}(\omega) < v_n\}\right)$$

$\forall v_1, v_2, ..., v_n \in R$, $\forall n \in N^*$,

Which fulfills the compatibility condition:

$$\lim_{x_i \to \infty} F_{t_1,t_2,...,t_n}(x_1, ...x_{i-1}, x_i, x_{i+1}..., x_n) = F_{t_1,...,t_{i-1},t_{i+1}...,t_n}(x_1, ...x_{i-1}, x_i, x_{i+1}..., x_n), \ \forall i = \overline{1,n}$$

The chronological series obtained after the statistic observation is $\{v_t | t \in T_c\}$, where $T_c \subseteq [0,\infty)$, T_c finite multitude, that is the result of a temporal observation of an economic variable.

Stochastically speaking, the taking over of the chronological series is made on the probability borelian field (Ω, K, P) associated to the process determining the economic variable.

We define the time series as the stochastic process $(V_t(\bullet))_{t\in T}$ over (Ω, K, P) and $T_c \subset T \subseteq R$, having the property of being $\omega \in \Omega$, so that $V_t(\omega) = v_t$, $\forall t \in T_c$.

The econometric pattern of the chronological series noted $(V_t)_{t\in T}$ is defined by the time series $(V_t(\bullet))_{t\in T}$ and the time series achievement is a family of values $(V_t)_{t\in T}$.

The Time Independence of the Chronological Series

The chronological series with an evolution independent of time or stationary is the series which has neither tendency nor circularity, nor seasonality, thus it is not influenced by the number of observations (which can be small or big) and neither by the beginning moment of the observation. The problem of the predictions for this chronological series is simplified, because a similar set of factors constantly acts on the economic variable.

Econometrically speaking, we deal with the issue of moulding the stationarity of a chronological series by defining the stationary notion for the time series which are econometric patterns for the chronological series.

We define time series $(V_t)_{t\in T}$ strictly stationary if:

$$F_{V_{t_1},\dots,V_{t_n}}(v_{t_1},v_{t_2},\dots,v_{t_n}) = F_{V_{t_1+h},\dots,V_{t_n+h}}(v_{t_1},v_{t_2},\dots,v_{t_n})$$

$\forall\{t_1,..,t_n\}\neq\{t_{1+h},..,t_{n+h}\}\neq\varnothing$ and all value sequences $(v_{t_1},v_{t_2},\dots,v_{t_n})$ from the domain of the aleatory variable values V_t.

We mention that the indexes $t_1,\dots,t_n$ are not consecutive; if the time series is strictly stationary then the repartition function of an aleatory variable is the same for any value from the indexes multitude and the common repartition depends only on the distance among the elements of the indexes multitude and not on their actual values.

If $(V_t)_{t\in T}$ is a strictly stationary time series then the average and the variant of the variable V_t are constant, the same for any *t*.

We call a time series $(V_t)_{t\in T}$ weakly stationary or stationary if the following *conditions* are fulfilled:

a. $M(V_t)=c,\ \forall t\in T,\ c\in R$;

b. $D^2(V_t)<\infty,\ \forall t\in T$;

c. the covariant matrix of $(V_{t_1},V_{t_2},\dots,V_{t_n})$ coincides with that of $(V_{t_1+h},V_{t_2+h},\dots,V_{t_n+h})$ for $\forall\{t_1,\dots,t_n\}\neq\varnothing$ and finite and all h for which $\{t_1,..,t_n,t_{1+h},..,t_{n+h}\}\subset T$.

h ‘s feature is to take those values for which the indexes multitude $\{t_1,..,t_n,t_{1+h},..,t_{n+h}\}$ is contained in *T* we note it with p_0.

Observations:

1. the constant c from the definition of the weakly stationary (a) by convention can be considered as being zero.
2. the covariant matrix is a time function only among the observations.

Given condition (c) and observation 2. we can say that a weakly stationary series is *stationary in covariant*. The selfcovariant function or the covariant function of the time series $(V_t)_{t\in T}$ stationary is noted and defined as follows:

$$\gamma : R\rightarrow R,\ \gamma(h)=\text{cov}(V_t,V_{t+h})=\text{cov}(V_{t+h},V_t)=M(V_t,V_{t+h})$$

where $M(V_t) = 0$ and h has the feature p_0.

The *consequences* of the previous definition are as follows:

a) $\gamma(0) = \text{cov}(V_t, V_t) = M\left(V_t^2\right) - \left[M\left(V_t\right)\right]^2 = D^2\left(V_t\right), \forall t \in T$;

b) $\gamma(-h) = \gamma(h), \forall h$ with the property p_0;

c) the covariant function of a stationary time series is non negatively defined.

The selfcovariant function of a stationary time series depends on the data measure which forms the chronological series and consequently to compare the time series features we must eliminate the measure by introducing a new function called self correlation function or correlation function.

The time series correlation function $\left(V_t\right)_{t \in T}$ is noted and defined as:

$$\rho : R \to R,\ \rho(h) = \frac{\gamma(h)}{\gamma(0)}.$$

The correlation coefficient between two aleatory varaiables defined on the same probability field justifies the notation used for the selfcorrelation function and $\left|\rho(h)\right| \leq 1, \forall h$ with the feature p_0.

The selfcorrelation function diagram can be achieved using different softwares, such as Mathematics, Statistics etc. and it is called corelogram.

The Stochastic Equation of the Selfregressive Series

The stochastic equation of a chronological series $\left(V_t\right)_{t \in T} T = Z$, non homogeneous linear with differences of order n with the constant coefficients is :

$$a_0 V_t + a_1 V_{t-1} + a_2 V_{t-2} + ... + a_n V_{t-n} = b_t,\ t \in \{n, n+1, ...\}, n \in N^* \quad (1)$$

where $a_n \neq 0, a_i \in R \forall i \in N^*$, b_t non-zero function of t.

for $b_t = 0$ equation (1) is called homogeneous stochastic equation with differences of order n with constant coefficients.

In the stochastic equations the terms are aleatory variables of some stochastic processes and the regressive differences are functions on multitude *Z*.

The chronological series $\left(V_t\right)_{t \in T}$ is associated to a real function family corresponding to a stochastic process and consequently to solve the homogeneous and linear equation with finite differences of order n with constant coefficients, we attach the characteristic equation:

$$a_0 r^t + a_1 r^{t-1} + ... + a_n r^{t-n} = 0 \quad (2)$$

whose roots are in *C*.

The chronological series $(V_t)_{t\in T}$ is called selfregressive order p if $T = Z$ and there are real coefficients a_i, $i = \overline{0, p}$, $a_0 \neq 0$, $a_p \neq 0$, so that $a_0V_t + a_1V_{t-1} + a_2V_{t-2} + ... + a_pV_{t-p} = e_t$, $t \in Z$, $p \in N^*$ şi $(e_t)_{t\in Z} \in ZA(0,\sigma^2)$.

We consider AR(p) the multitude of all selfregressive chronological series of order p.

If there is a row of numbers $(a_n)_{n\in N}$ and $a_0 = 1$ so that

$$a_0V_t + a_1V_{t-1} + a_2V_{t-2} + ... + a_nV_{t-n} = e_t,\ t \in T = Z$$

then $(V_t)_{t\in T}$ is self regressive of indefinite order.

For $(V_t)_{t\in T} \in \mathrm{AR}(1)$ and $T = Z$ the self regressive chronological series presents itself like:

$$a_0V_t + a_1V_{t-1} = e_t,\ a_0 \neq 0, a_1 \neq 0 \text{ şi } (e_t)_{t\in Z} \in ZA(0,\sigma^2) \tag{3}$$

Equation (3) is a linear stochastic equation, non homogeneouus with differences of order 1 and constant coefficients, having attached the homogeneous equation:

$$a_0V_t + a_1V_{t-1} = 0,\ a_0 \neq 0, a_1 \neq 0 \tag{4}$$

And the characteristic equation in its formal form is:

$$a_0r + a_1 = 0,\ a_0 \neq 0, a_1 \neq 0$$

With the solution: $r = -\frac{a_1}{a_0} = -a,\ a \neq 0$.

For $|a| < 1$ şi $M_2(V_t) < M < \infty,\ \forall t \in Z$ the series $(V_t)_{t\in Z}$ is a mobile average of indefinite order with $V_t = \sum_{i}^{\infty} (-a)^i e_{t-i}$.

The covariant function of the chronological series is defined as follows:

$$\gamma(h) = M(V_t \cdot V_{t-h}) = M\left(\left(\sum_{i=0}^{\infty}(-a)^i e_{t-i}\right)\left(\sum_{j=0}^{\infty}(-a)^j e_{t-h-j}\right)\right) = \sum_{i=h}^{\infty}(-a)^i(-a)^{i-h} M\left(e_{t-i}^2\right) =$$

$$= \sum_{i=n}^{\infty}(-a)^i(-a)^{i-h} D^2\left(e_{t-i}\right) = \sigma^2 \sum_{i=h}^{\infty}(-a)^i(-a)^{i-h},\ h \in N\ .$$

Considering that $\frac{a_1}{a_0} = a$, equation (3) is equivalent to $V_t + aV_{t-1} = e_t$, $a \neq 0$, from where we get:

$$V_t \cdot V_{t-h} + aV_{t-1} \cdot V_{t-h} = e_t \cdot V_{t-h} \tag{5}$$

Using the average operator, relation (5) becomes:

$M(V_t \cdot V_{t-h}) + aM(V_{t-1} \cdot V_{t-h}) = M(e_t \cdot V_{t-h})$

From where we obtain:

$$\gamma(h) + a\gamma(h-1) = \begin{cases} \sigma^2, h = 0 \\ 0, \quad h \geq 1 \end{cases} \tag{6}$$

From relation (6) we deduce that any $h \geq 1, h \in N$ the covariant functionis solution of the homogeneous stochastic equation $V_t + aV_{t-1} = 0$ and for $\gamma(0) \neq 0$ we get $\rho(h) + a\rho(h-1) = 0, h \in N$ * with the initial conditions $\rho(0) = 1$ and $\rho(1) = -a$. The solution for this equation is the correlation function $\rho(h) = (-a)^h$, $h \in N$.

The representation theorem of the self regressive series

Be $(V_t)_{t \in T} \in AR(p)$ so that:

1) $\sum_{i=0}^{p} a_i V_{t-i} = e_t$, $t \in Z$, $p \in N^*$, $a_0 = 1$, $a_p \neq 0$;
2) $M_2(V_t) < M$, $\forall t \in Z$, $M \in R^*$;
3) the roots of the characteristic equation attached to the non homogeneous stochastic equation 1) have the module strictly subunitary.

In conclusion:

1') the solution of the stochastic equation 1) is: $V_t = \sum_{i=0}^{\infty} b_i \cdot e_{t-i}$ where the coefficients $(b_i)_{i \in N}$ verify the next system of homogeneous equations with finite differences:

$b_i + a_1 b_{i-1} + ... + a_p b_{i-p} = 0, \ \forall i \in \{p, p+1, ...\}$

Having the initial conditions : $b_0 = 1,\ b_i + \sum_{j=1}^{i} a_j b_{i-j} = 0,\ \ \forall i = \overline{1, p-1}$;

2') the chronological series is stationary in covariance.

Proof. (1') following the ideas from *Introduction to statistical time series* by Fuller A.W., we assume that $(V_t)_{t \in T} \in AR(p)$ is defined on the borelian field (Ω, K, P) and the homogeneous stochastic equation attached is:

$$\sum_{i=0}^{p} a_i \cdot V_{t-i} = 0,\ t \in Z,\ p \in N^* \tag{7}$$

And the corresponding characteristic equation is:

$$\sum_{i=0}^{p} a_i \cdot r^{t-i} = 0,\ \ p \in N^*,\ a_0 = 1 \tag{8}$$

With the roots r_i, $i = \overline{0, p}$ şi $|r_i| < 1,\ \forall i = \overline{0, p}$.

From relation (7) for $a_0 = 1$ we get: $V_t = -\sum_{i=1}^{p} a_i V_{t-i}$ and we consider the system:

$$\begin{cases} V_t = -\sum_{i=1}^{p} a_i V_{t-i} \\ V_{t-1} = V_{t-1} \\ V_{t-(p-1)} = V_{t-(p-1)} \end{cases}$$

having the matrix form $Y_t = AY_{t-1} + \varepsilon_t$, $t \in Z$, as $Y_{t-1} = AY_{t-2} + \varepsilon_{t-1}$ it results $Y_t = A^2 Y_{t-2} + A\varepsilon_t + \varepsilon_t$, so:

$$Y_t = A^n Y_{t-n} + \sum_{i=0}^{n} A^i \varepsilon_{t-i}, \; t \in Z, \; n \in N^* \tag{9}$$

where $Y_t = \begin{pmatrix} V_t \\ V_{t-1} \\ V_{t-(p-1)} \end{pmatrix}$, $A = \begin{pmatrix} -a_1 & -a_2 & -a_3 & \dots & -a_{p-1} & -a_p \\ 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 & 0 \end{pmatrix}$, $\varepsilon_t = \begin{pmatrix} e_t \\ 0 \\ \dots \\ 0 \end{pmatrix}$.

For the matrices from relation (9) the following notations are used:

$$A^k = \left(a_{ij}^k\right), \; k \in N^*, \; i, j \in \{1, 2, \dots, p\}$$

$$A^i = \left(a_{11}^i\right) = b_i, \; i = \overline{0, p}$$

and equation (9) becomes:

$$V_t = \sum_{j=1}^{p} a_{1j}^n V_{t-n-j} + \sum_{i=0}^{n-1} b_i e_{t-i} \tag{10}$$

The characteristic polynominal of matrix A is

$$\left|A - \lambda I_p\right| = \begin{vmatrix} -a_1-\lambda & -a_2 & -a_3 & \dots & -a_{p-1} & -a_p \\ 1 & -\lambda & 0 & \dots & 0 & 0 \\ 0 & 1 & -\lambda & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 & -\lambda \end{vmatrix} =$$

$$= (-a_1 - \lambda) \cdot \begin{vmatrix} -\lambda & 0 & \cdots & 0 \\ 1 & -\lambda & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & -\lambda \end{vmatrix} - \begin{vmatrix} -a_2 & -a_3 & \cdots & -a_p \\ 1 & -\lambda & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & -\lambda \end{vmatrix} =$$

$$= (-a_1 - \lambda) \cdot (-\lambda)^{p-1} - (-a_2) \cdot (-\lambda)^{p-2} + \begin{vmatrix} -a_3 & \cdots & -a_p \\ 1 & \cdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \cdots & -\lambda \end{vmatrix} =$$

$$= \dots = (-1)^p \sum_{i=0}^{p} a_i \lambda^{p-i}, \quad a_0 = 1.$$

Cayley-Hamilton theorem for matrix A is:

$$A^p + a_1 A^{p-1} + a_2 A^{p-2} + \dots + a_{p-1} A + a_p I_p = O_p,$$

From where we get by multiplying with A^{i-p}:

$$(A^{i} + a_1 A^{i-1} + a_2 A^{i-2} + \ldots + a_{p-1} A^{i-p+1} + a_p A^{i-p} = O_p ,\; i \geq p \qquad (11)$$

According to relation (11) we deduce that b_i verifies the conditions from (1'), and a_{1j}^{k} verifies the equation:

$$a_{1j}^{k} + a_1 a_{1j}^{k-1} + a_2 a_{1j}^{k-2} + \ldots + a_{p-1} a_{1j}^{k-p+1} + a_p a_{1j}^{k-p} = O_p ,\; \forall k \geq p \text{ for } j \in \{1,2,\ldots,p\}, n \geq p \qquad (12)$$

From relation (12) we get:

$$a_{1j}^{k} = -a_1 a_{1j}^{k-1} - a_2 a_{1j}^{k-2} - \ldots - a_{p-1} a_{1j}^{k-p+1} - a_p a_{1j}^{k-p},\; \forall j \in \{1,2,\ldots,p\}$$

therefore there is $c > 0$ and $\lambda \in (M,1)$ where $M = \max_{0 \leq i \leq 1} \{|\lambda_i|, \det(A - \lambda_i I_p) = 0\}$ so that

$\left|a_{1j}^{k}\right| < c\lambda^{k}$, $\forall j \in \{1,2,\ldots,p\}$ and any $k \geq p$.

Using the relation (10) and the average operator we deduce that:

$$M\left(\left(V_t - \sum_{i=0}^{n-1} b_i e_{t-i}\right)^2\right) = M\left(\left(\sum_{j=1}^{p} a_{1j}^{n} V_{t-n-j}\right)^2\right) \leq \left(c \cdot p \cdot \lambda^{n}\right)^2 \cdot M \xrightarrow{n \to \infty} 0$$

$$M\left(\left|V_t - \sum_{i=0}^{n-1} b_i e_{t-i}\right|\right) = \sum_{j=1}^{p} \left|a_{1j}^{n}\right| \cdot M\left(V_{t-n-j}\right) \leq c \cdot p \cdot \lambda^{n} M^{\frac{1}{2}}$$

so according to the dominated convergency theorem of Lebesque we get $V_t = \sum_{i=0}^{\infty} b_i \cdot e_{t-i}$ what was supposed to be demonstrated.

3') As $|b_i| = \left|a_{11}^{i}\right| < c \cdot \lambda^{i}$, $\forall i \geq p$ it results that $\sum_{i=0}^{\infty} b_i$ is absolutely convergent, so the chronological series $(V_t)_{t \in T} \in AR(p)$ is stationary in covariance according to Beppa Levi theorem.

Consequence. If the chronological series $(V_t)_{t \in T}$ verifies the representation theorem of the self regressive series, then the covariance function attached verifies the following relations:

$$\gamma(0) + a_1 \gamma(1) + \ldots + a_p \gamma(p) = \sigma^2$$

$$\gamma(h) + a_1 \gamma(h-1) + \ldots + a_p \gamma(h-p) = 0,\; \forall h \in N^*.$$

Justification. According to relation 1) from the theorem we can write that $V_{t-h} e_t = \sum_{i=0}^{p} a_i V_{t-h} V_{t-i}$ so:

$$M(V_{t-h} \cdot e_t) = M\left(\sum_{i=0}^{p} a_i V_{t-h} V_{t-i}\right) = \sum_{i=0}^{p} a_i M(V_{t-h} V_{t-i}) = \sum_{i=0}^{p} a_i \gamma(h-i) = \begin{cases} \sigma^2, & h = 0 \\ 0, & h \geq 1 \end{cases}.$$

The Stochastic Equation of the Mobile Average

The chronological series $(V_t)_{t\in T}$ is a mobile average of order n, if $T = Z$ and

$$\sum_{i=0}^{p} a_i e_{t-i} = V_t \tag{13}$$

where $n \in N^*$, $a_i \in R$, $\forall i \in \{0,1,\ldots,n\}$, $a_0 = 1$, $a_n \neq 0$ and $(e_t)_{t\in Z} \in ZA(0, \sigma^2)$.

We note $MA(n)$, $n \in N^*$ the multitude of all mobile averages of order n.

If there is an absolutely convergent numerical series $\sum_{i=0}^{\infty} a_i$ with $a_0 = 1$ so that relation (13) take place, then the series $(V_t)_{t\in T}$ is a mobile average of indefinite order.

If $(V_t)_{t\in T}$ is a finite mobile average then yhis is called inverted, if it can be represented as a self regressive series of indefinite order.

A mobile average of indefinite order $(V_t)_{t\in T}$ with $V_t = \sum_{i=0}^{\infty} a_i \cdot e_{t-i}$ and $\sum_{i=0}^{\infty} a_i$ an absolutely covergent series, according to Beppo-Levi theorem it is stationary.

The chronological series $(V_t)_{t\in T} \in MA(n)$, $T = Z$ with:

$$V_t = \sum_{i=0}^{n} a_i e_{t-i} \text{ where } n \in N^*,\ a_0 = 1,\ a_n \neq 0 \text{ and } a_i \in R,\ \forall i = \overline{0,1} \text{ and } (e_t)_{t\in Z} \in ZA(0, \sigma^2) \tag{14}$$

is inverted if the roots of the characteristic equation corresponding to (14) are strictly lower than 1.

From relation (14) under these circumstances we get: $e_t = \sum_{j=0}^{\infty} b_j \cdot V_{t-j}$, $t \in Z$, where the coefficients $(b_j)_{j\in N}$ verify the next system of homogeneous equations with finite differences:

$$b_i + a_1 b_{i-1} + \ldots + a_p b_{i-p} = 0,\ \forall i \in \{n, n+1, \ldots\}$$

having the following initial conditions:

$$\begin{cases} b_0 = 1 \\ b_1 = -a_1 \\ b_2 = -a_1 b_1 - a_2 \\ \quad \ldots\ldots \\ b_{n-1} = -a_1 b_{n-2} - a_2 b_{n-3} - \ldots - a_{n-1} \end{cases}$$

Conclusions

Most economic processes are unstationary, thus the analysis of these processes is made with the appropriate methods. The express use of stationary methods is made in special occasions, therefore it is necessary to study the modeling of the stationarity of a chronological series, so that the methods used in analyzing and forecasting will be more efficient.

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Studiul proprietății de staționaritate a seriilor cronologice

Rezumat

Construcția modelelor statistice sau econometrice ale seriilor cronologice se face şi cu ajutorul proceselor stochastice. În acest articol se prezintă o problemă de modelare a proprietății de staționaritate a unei serii cronologice folosind ecuația stochastică a unei serii cronologice şi ecuația stochastică a mediei mobile.