

# The Construction of the Membership Functions in the Fuzzy Measuring of Poverty

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## Abstract

*This paper gives an insight to the fuzzy theory that has been increasingly used lately in the analysis of poverty and deprivation seen as a multi-dimensional condition. A multi-dimensional approach in measuring and analysing poverty involves both monetary aspects such as incidence and intensity of low income and diverse non-monetary aspects such as lack of access to other resources, facilities, social interactions and even individual attributes determining the life-style.*

*We consider that it is more useful to view poverty or deprivation as a matter of degree, giving a quantitative expression to its intensity for individuals in different dimensions. This paper regards only the monetary dimension of the poverty, and is focused on the methodological aspects related to the construction of the membership functions and, by extension to the construction of the fuzzy monetary measure. In the last part of this paper we determine, based on the methodological aspects presented, the membership functions to poverty for the households distributed by deciles of total income per person, for years 2003 and 2006 in Romania.*

**Key words:** *fuzzy sets, membership-functions, fuzzy monetary measures*

**JEL Classification:** *C02, C65, I32*

## Introduction

Most of the methods used in the poverty measuring suffer of two main limitations: they refer to only one dimension of the phenomenon concerned, in general the monetary one, and they divide the population in two distinct categories: poor and non-poor by means of the so-called poverty threshold. We will focus in this paper only on the second limitation of the conventional approaches. Regarding poverty as a phenomenon that affects population from the severe poverty, meaning distinct material hardship, to substantial welfare, in many different ways, it is clear that dividing the population only in two classes removes the nuances that exist between these two extremes and causes an important loss of information. From this point of view, the researchers agreed that it is more appropriate to consider poverty as a matter of degree rather than an attribute that is simply present or simply absent for individuals in the population. This concept of “fuzziness” applied in the measuring of the poverty, was developed for the first time by Cerioli and Zani (1990) who based their researches on the mathematical theory of fuzzy sets introduced by Zadeh (1965).

In this paper we will focus on the latest approaches on fuzzy analysis of poverty, mainly on the choice of the membership functions (m.f.), regarding quantitative specification of individuals’

degree of poverty. The m.f. may be extended to the set of all subpopulations of the main population. The mapping obtained in this way is called the degree of poverty and is a fuzzy (monotone) measure with respect to a specified set inclusion preorder. We shall base our further presentation on the mathematical theory of fuzzy sets.

Let  $(\Omega, \mathcal{F})$  be a measurable space where the set  $\Omega$  is the so-called population set and  $\mathcal{F}$  is a  $\sigma$ -algebra on  $\Omega$ , or the set of all subsets of  $\Omega$  such that:

$$\Omega \in \mathcal{F} \quad (1)$$

$$A \in \mathcal{F} \text{ implies } A^C \in \mathcal{F} \quad (2)$$

$$(A_n)_n \subset \mathcal{F} \text{ implies } \bigcup_{n=1}^{\infty} A_n \in \mathcal{F} \quad (3)$$

According to the definition given by Zadeh (1965), a fuzzy set  $S$  in  $\Omega$  is characterized by a membership function  $\mu_S : \Omega \rightarrow [0,1]$ , where  $\mu_S(i)$  represents the degree of membership of  $i$  in  $S$ . The nearer the value of  $\mu_S(i)$  to unity, the higher the degree of membership of  $i$  in  $S$ . Thus  $\mu_S(i) = 0$  means that  $i$  does not belong to  $S$ , whereas  $\mu_S(i) = 1$  means that  $i$  belongs to  $S$  completely.

**Definition 1.** Let  $(\Omega, \mathcal{F})$  be a measurable space. A function  $m : \mathcal{F} \rightarrow [0, \infty)$  is a fuzzy measure (monotone measure) if:

$$m(\Phi) = 0 \quad (4)$$

$$A, B \in \mathcal{F}, A \subseteq B, \text{ then } m(A) \leq m(B) \quad (5)$$

Trillas and Alsina (1999) gave a general definition of a fuzzy measure in the case that a preorder relation is defined on the set  $\Omega$ . When a characteristic, (in our case the degree of poverty), needs to be measured on the elements of a set  $\Omega$ , a preorder relation that allows to stand that “ $i$  shows the characteristic less than  $j$  shows it” ( $i \prec j$ ), for all  $i, j$  in  $\Omega$ , is necessary to be set.

The preorder relation (reflexive and transitive relation) is denoted by  $i \prec j$ .

**Definition 2.** Let  $\prec$  be a preorder, for which  $min$  is the minimal element in  $\Omega$  and  $max$  is the maximal element in  $\Omega$ . Then the fuzzy  $\prec$ -measure is a function  $m : \Omega \rightarrow [0,1]$  that verifies the following conditions:

$$m(min) = 0 \quad (6)$$

$$m(max) = 1 \quad (7)$$

$$\text{If } i \prec j \text{ then } m(i) \leq m(j) \quad (8)$$

From now on we will have in mind the set  $\Omega$  as the population to be investigated and the fuzzy set  $S$  as the poor population. Let us consider  $m(i)$  the degree of poverty of the individual  $i$ .

Construction of the Monetary Membership Function (m.f.)

Let  $\Omega = \{1, 2, \dots, m\}$ . For  $S$  fuzzy set of  $\Omega$ , with the previous remarks we may now consider the membership function:

$$\mu_S : \Omega \rightarrow [0,1] \quad (9)$$

$$\mu_S(i) = m(i) \text{ for any individual } i \in \Omega \quad (10)$$

And to extend this membership function to  $\mu_S : \mathcal{F} \rightarrow [0,1]$ , defined as:

$$\mu_S(A) = \frac{\sum_{i \in A} \mu_S(i)}{\text{card}(A)} \text{ for any } A \in \mathcal{F} \tag{11}$$

We shall refer to  $S$  as to the set of poor persons, and to  $\mu_S(A)$  as to the overall membership degree of  $A$  to the fuzzy set  $S$ .

We shall prove that the degree of poverty  $\mu_S : \Omega \rightarrow [0,1]$  is a fuzzy  $\prec$ -measure, in the sense of definition 2. In our case, the preorder  $\prec$  becomes:  $i \prec j$  is equivalent to “individual  $i$  is less poor than individual  $j$ ” ( $\mu_S(i) \leq \mu_S(j)$ ). Moreover, if  $i$  is the minimal element of  $\Omega$  with respect to  $\prec$ , meaning that  $i$  is the least poor (the richest) individual in our population, then  $\mu_S(i) = 0$  and, for  $j$  the maximal element of  $\Omega$  with respect to  $\prec$  (poorest individual), we have  $\mu_S(j) = 1$ .

Moreover, if we consider the set inclusion preorder in the following way:  $A \prec B$  if  $\mu_S(A) \leq \mu_S(B)$ , then we may observe that the extended fuzzy  $\prec$ -measure  $\mu_S : \mathcal{F} \rightarrow [0,1]$  defined above is a monotone measure with respect to the specified set inclusion preorder. These statements are available for all three variants of membership functions described further on.

Let  $(\Omega, \mathcal{F}, P)$  be the probability space having the classical probability  $P : \mathcal{F} \rightarrow [0,1]$  defined as:  $P(A) = \frac{\text{card}(A)}{\text{card}(\Omega)}$  for any  $A \in \mathcal{F}$ .

Let  $y : \Omega \rightarrow [0,+\infty)$  be the random variable which assigns to each person  $i$  the equalised income of the person’s household,  $y(i)$ . Consider the individuals’ incomes sorted in increasing order, such that  $y_1$  is the income of the poorest individual.

Let  $F : [0,+\infty) \rightarrow [0,1]$  be the distribution function of income of the individual having income strictly greater than the poorest individual.

$$F(y(i)) = P(y \leq y(i) / y > y_1) \tag{12}$$

Denote  $w_i = P(y = y(i) / y > y_1)$  the share of the population having income equal to  $y(i)$  in the total population having income strictly greater than  $y_1$ .

$F(y(i))$  represents the proportion of individuals having an income strictly higher than the poorest person’s income and lower than the income of the person’s concerned,  $i$ .

### Variant I

According to Totally Fuzzy and Relative (T.F.R.) approach introduced by Cheli and Lemmi (1995), the membership function  $\mu_S : \Omega \rightarrow [0,1]$  is defined as follows:

$$\mu_S = 1 - F \circ y \tag{13}$$

$$\mu_S(i) = 1 - F(y(i)) = \left( 1 - \frac{\sum_j \omega_j / y_1 < y_j \leq y(i)}{\sum_j \omega_j / y_j > y_1} \right) \quad (14)$$

$$\mu_S(i) = \left( \frac{\sum_j \omega_j / y_j > y(i)}{\sum_j \omega_j / y_j > y_1} \right) \quad (15)$$

$1 - F(y(i))$  can also be interpreted as the proportion of the individuals less poor than the person concerned.

**Remark 1.** We observe that the membership function has value 1 for the poorest individual (the maximal element with respect to  $\prec$ ) and value 0 for the richest individual (the minimal element with respect to  $\prec$ ).

In order to facilitate comparison between the conventional and fuzzy measures, a parameter  $\alpha \geq 1$  is introduced.

$$\mu_S(i) = (1 - F(y(i)))^\alpha = \left( \frac{\sum_j \omega_j / y_j > y(i)}{\sum_j \omega_j / y_j > y_1} \right)^\alpha \quad (16)$$

**Remark 2.** We observe that for two individuals  $i \prec j$  ( $i$  is less poor than  $j$ ) we have  $y(i) \geq y(j)$  and, according to (16) we obtain that  $\mu_S(i) \leq \mu_S(j)$ . Moreover, if we have  $A = \{i_1, i_2, \dots, i_p\} \subset \Omega$  a subpopulation having the overall membership degree to poverty  $\mu_S(A)$ , then by adding to this subpopulation an extra individual  $\{i_{p+1}\}$  we have one of the following possibilities:

If the individual has the membership degree to poverty  $\mu_S(i_{p+1}) \geq \mu_S(A)$ , then the subpopulation  $A \cup \{i_{p+1}\}$  has the overall membership degree to poverty  $\mu_S(A \cup \{i_{p+1}\}) \geq \mu_S(A)$ . Indeed:

$$\mu_S(A \cup \{i_{p+1}\}) = \frac{1}{p+1} [\mu_S(i_1) + \dots + \mu_S(i_p) + \mu_S(i_{p+1})]$$

$$\mu_S(A \cup \{i_{p+1}\}) \geq \frac{1}{p+1} \left[ \mu_S(i_1) + \dots + \mu_S(i_p) + \frac{1}{p} (\mu_S(i_1) + \dots + \mu_S(i_p)) \right] = \mu_S(A)$$

We say that subpopulation  $A \cup \{i_{p+1}\}$  is more poor than the subpopulation  $A$ , or else  $A \prec A \cup \{i_{p+1}\}$ .

If the individual has the membership degree to poverty  $\mu_S(i_{p+1}) \leq \mu_S(A)$ , then the subpopulation  $A \cup \{i_{p+1}\}$  has the overall membership degree to poverty

$\mu_S(A \cup \{i_{p+1}\}) \leq \mu_S(A)$ . We say that subpopulation  $A \cup \{i_{p+1}\}$  is less poor than the subpopulation  $A$ , or else  $A \cup \{i_{p+1}\} \prec A$ .

**Remark 3.** Increasing the value of this exponent  $\alpha$  means giving more weight to the poorer end of income distributions. Large values of the m.f. would then be concentrated at that end, making the propensity to income poverty sensitive to the position of the poorer persons in the income distributions.

According to Cheli and Betti (1999) and Betti and Verma (1999),  $\alpha$  is chosen such that the mean of the membership function equals the head count ratio  $H$ . By the transport formula we obtain:

$$E[\mu_S] = \int_{\Omega} \mu_S dP = \int_{\Omega} (1 - F(y))^\alpha dP = \int_R (1 - F)^\alpha dP \circ y^{-1} = \int_{[0,1]} (1 - F)^\alpha dF \tag{17}$$

$$E[\mu_S] = \int_{[0,1]} (1 - F)^\alpha dF = \frac{1}{1 + \alpha} \tag{18}$$

$$\frac{1}{1 + \alpha} = H \tag{19}$$

$$\alpha = \frac{1}{H} - 1 \tag{20}$$

**Variant II**

A refined version of the formula (15), given by Betti and Verma (1995), define the membership function taking into account the cumulative share of the income earned by all individuals less poor than the people concerned:

$$\mu_S = [1 - L \circ F \circ y]^\alpha \tag{21}$$

$$\mu_S(i) = [1 - L(F(y(i)))]^\alpha = \left( \frac{\sum_j \omega_j y_j / y_j > y(i)}{\sum_j \omega_j y_j / y_j > y_1} \right)^\alpha \tag{22}$$

Where  $L$  represents the Lorenz curve of income.  $1 - L(F(y(i)))$  represents the share of the total equalised income received by all individuals less poor than the people concerned. It varies from 1 for the poorest, to 0 for the richest individual.  $1 - L(F(y(i)))$  is a more sensitive indicator of the real disparities in income, compared to  $1 - F(y(i))$ .

**Remark 4.** The mean of  $\mu_S = [1 - L \circ F \circ y]$  is  $\frac{1 + G}{2}$  where  $G$  is the Gini coefficient of distribution.

Indeed,

$$E[\mu_S] = \int_{\Omega} (1 - L(F(y))) dP = \int_R (1 - L(F)) dP \circ y^{-1} = \int_{[0,1]} (1 - L(F)) dF \tag{23}$$

On the other hand:

$$G = \frac{\int_{[0,1]} (F - L(F)) dF}{\frac{1}{2}} = 2 \int_{[0,1]} (F - L(F)) dF = 2 \int_{[0,1]} (1 - L(F)) dF + 2 \int_{[0,1]} (F - 1) dF$$

$$G = 2E[1 - L(F(y))] + 2 \int_{[0,1]} (F - 1) dF = 2E[1 - L(F(y))] - 1$$

$$E[1 - L(F(y))] = \frac{G + 1}{2} \quad (24)$$

Moreover, in the case of perfect distribution, meaning  $G = 0$ , we have  $L(F(y(i))) = F(y(i))$  for all  $i$ , hence  $E[1 - F(y)] = E[1 - L(F(y))] = \frac{1}{2}$ .

### Variante III

An Integrated Fuzzy and Relative approach (I.F.R.) that combine the T.F.R. of Cheli and Lemmi (1995) and Betti and Verma (1999), was set up by Betti, Cheli, Lemmi and Verma (2005).

The proposed membership function considers both the share of individuals who are less poor than the people concerned ( $1 - F(y(i))$ ), and also the share of the total equalised income received by all individuals less poor than the person concerned ( $1 - L(F(y(i)))$ ).

The m.f. is defined as:

$$\mu_S = [1 - F \circ y]^{\alpha-1} [1 - L \circ F \circ y] \quad (25)$$

$$\mu_S(i) = [1 - F(y(i))]^{\alpha-1} [1 - L(F(y(i)))] = \left( \frac{\sum_j \omega_j / y_j > y(i)}{\sum_j \omega_j / y_j > y_1} \right)^{\alpha-1} \left( \frac{\sum_j \omega_j y_j / y_j > y(i)}{\sum_j \omega_j y_j / y_j > y_1} \right) \quad (26)$$

Parameter  $\alpha$  is chosen such that the mean of the m.f. is equal to the conventional head count ratio  $H$ . The m.f.  $\mu_S(i) = [1 - F(y(i))]^{\alpha-1} [1 - L(F(y(i)))]$  may be expressed in terms of the generalised Gini coefficient  $G_\alpha$  with  $\alpha \geq 1$ .

Where  $G_\alpha$  is defined in the continuous case as following:

$$G_\alpha = \alpha(\alpha + 1) \int_{[0,1]} (1 - F)^{\alpha-1} (F - L(F)) dF \quad (27)$$

We may assign the following interpretation:  $G_\alpha$  weighs the distance between the line of perfect equality and Lorenz curve  $F - L(F)$ , by a function of the individual's position in the income distribution  $(1 - F)^{\alpha-1}$ , giving more weight to its poorer end.

Starting from (26) we may obtain the mean of the m.f. given in (30)

$$G_\alpha = \alpha(\alpha + 1) \int_{[0,1]} (1 - F)^{\alpha-1} (F - L(F) + 1 - 1) dF \quad (28)$$

$$G_{\alpha} = \alpha(\alpha + 1) \int_{[0,1]} (1-F)^{\alpha-1} (1-L(F)) dF + \alpha(\alpha + 1) \int_{[0,1]} (1-F)^{\alpha-1} (F-1) dF$$

Knowing that:

$$E[\mu_S] = \int_{\Omega} (1-F(y))^{\alpha-1} ((1-L(F(y)))) dP = \int_{[0,1]} (1-F)^{\alpha-1} (1-L(F)) dF \quad (29)$$

We obtain:

$$G_{\alpha} = \alpha(\alpha + 1)E[\mu_S] - \alpha(\alpha + 1) \int_{[0,1]} (1-F)^{\alpha} dF = \alpha(\alpha + 1)E[\mu_S] - \alpha \quad (30)$$

Finally, from  $E[\mu_S] = \frac{\alpha + G_{\alpha}}{\alpha(\alpha + 1)} = H$  we get the value of the parameter  $\alpha$ .

## An Example for Computing the Membership Function for Romania

The further construction of the monetary membership function (m.f.) for Romania, is based on the methodological aspects presented above (all the three variants). Our aim is to compute the membership functions to the poverty of households distributed by deciles. Our statistical unit is in this first case the household and the statistical population is the set of all households in Romania. The first decile contains the most poor 10% households of the total households (taking into account the total income per person), while the tenth decile contains the least poor 10% households. We shall use in our analysis the data provided by the National Institute of Statistics in the Romanian Statistical Yearbook 2007 and 2004, meaning the distribution of households by decile of total income per person, in 2003 and 2006. We have calculated the m.f. according to formulas (16), (22), and (26). The value of the parameter  $\alpha$  for the first two variants was chosen by applying formula (20), knowing that the national poverty rate was 25,2% in 2003 and 13,8% in 2006. For the third variant  $\alpha$  was chosen such that the mean of the membership function is equal to the national head cont ratio. We have obtained two values for each year, (corresponding to variants I and II, respectively variant III ) These values are: for 2003  $\alpha_1 = \alpha_2 = 6,24$ ,  $\alpha_3 = 11$ , and for year 2006  $\alpha_1 = \alpha_2 = 3$ , respectively  $\alpha_3 = 3,95$ .

The calculated values for the membership functions, in the three variants are in Table 1 and Table 2. These values reflect the degree of poverty of the households belonging to each decile.

**Table 1.** Distribution of households by deciles of total income per person, in 2006

	Decile 1	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10
Monthly income per household	634,650	765,520	858,460	999,890	1095,150	1193,780	1400,260	1641,910	2006,090	3266,520
households (percent)	0,010	0,010	0,010	0,010	0,010	0,010	0,010	0,010	0,010	0,010
m.f.1	1,000000	0,888889	0,777778	0,666667	0,555556	0,444444	0,333333	0,222222	0,111111	0,000000
m.f.2	1,000000	0,688944	0,441014	0,251107	0,127047	0,054931	0,017348	0,003187	0,000160	0,000000
m.f.3	1,000000	0,290124	0,071067	0,013902	0,002013	0,000189	0,000009	0,000000	0,000000	0,000000

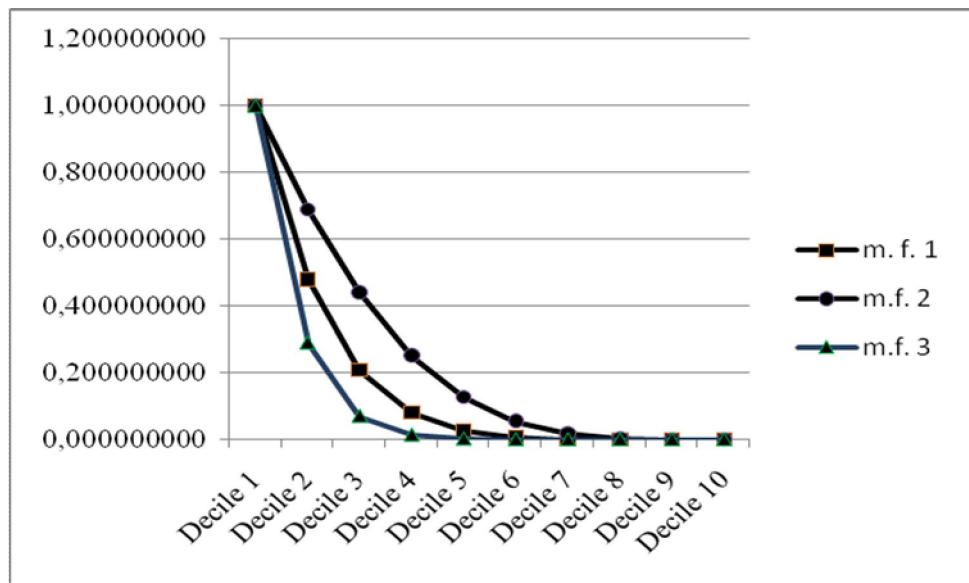
*Income intervals are expressed in the prices of January 2006.*

**Table 2.** Distribution of households by deciles of total income per person, in 2003

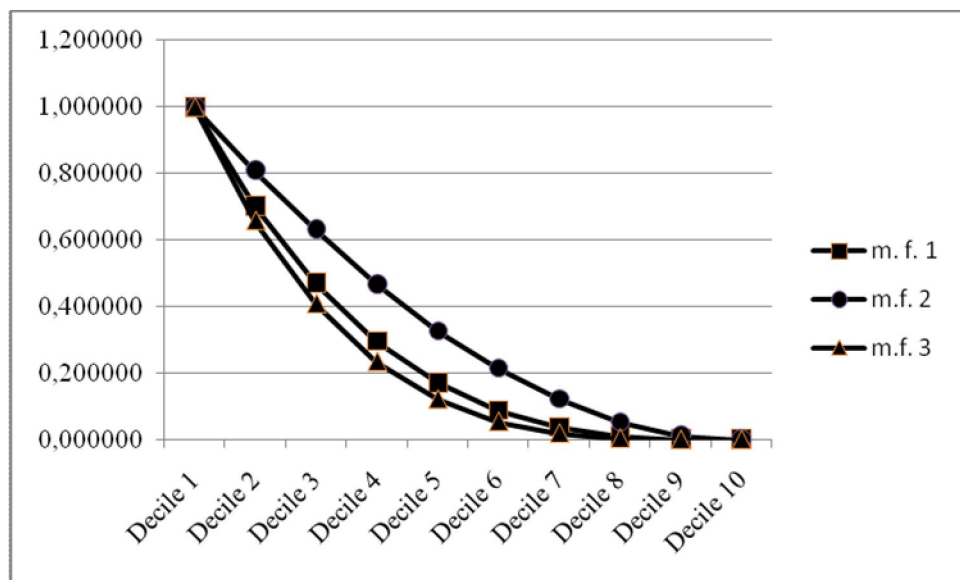
	Decile 1	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10
Monthly income per household	4317529	5111419	5531134	6167950	6607315	6798024	7651162	8973264	11113088	17235612
households (percent)	0,01000	0,01000	0,01000	0,01000	0,01000	0,01000	0,01000	0,01000	0,01000	0,01000
m.f.1	1,00000	0,70233	0,47051	0,29630	0,17147	0,08779	0,03704	0,01097	0,00137	0,00000
m.f.2	1,00000	0,80961	0,63264	0,46805	0,32644	0,21399	0,12230	0,05360	0,01205	0,00000
m.f.3	1,00000	0,65845	0,40902	0,23476	0,12158	0,05468	0,01942	0,00446	0,00035	0,00000

Income intervals are expressed in the prices of January 2006.

The figures of the membership functions are illustrated in Figure 1 and Figure 2.



**Fig. 1.** Graphic of the membership functions m.f.1, m.f.2, and m.f.3 for 2006



**Fig. 2.** Graphic of the membership functions m.f.1, m.f.2, and m.f.3 for 2003

We may observe that for both years 2003 and 2006, the graphic of m.f.1 is situated between the curves of m.f.2 and m.f.3 . According to these graphics, the representation of m.f. 3 gives us a



more optimistic view on the poverty than the other two variants (m.f.1. reflects the most pessimistic approach). As compared to 2003, year 2006 shows us a more improved situation, illustrated by the position of the m.f.3. (2003) curve above the curve m.f.3 (2006) for each of the ten deciles, as Figure 3 shows.

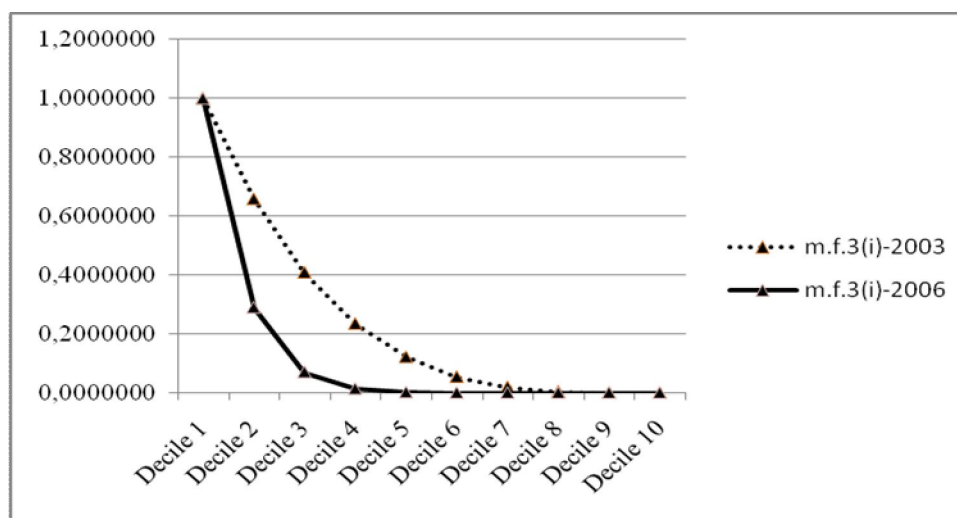


Fig. 3. Graphic of the membership function m.f.3 for 2003 and 2006

Similarly, we may analyse the degree of poverty of the individuals belonging to deciles, considering as statistical unit the individual and as statistical population the whole population of Romania. We shall assume that the individuals belonging to households of the same decile have the same degree of poverty, and consequently the same membership function.

We have calculated the m.f. according to formulas (16), (22), and (26). The value of the parameter  $\alpha$  for the first two variants was chosen by applying formula (20), knowing that the national poverty rate was 25,2% in 2003 and 13,8% in 2006. For the third variant  $\alpha$  was chosen such that the mean of the membership function is equal to the national head cont ratio. We have obtained two values for each year, (corresponding to variants I and II, respectively variant III). These values are: for 2003  $\alpha_1 = \alpha_2 = 6,246$ ,  $\alpha_3 = 10$ , and for year 2006  $\alpha_1 = \alpha_2 = 3$ , respectively  $\alpha_3 = 3,73$ . The calculated values for the membership functions, in the three variants are in Table 3 and Table 4. These values reflect the degree of poverty of the persons belonging to households distributed by deciles.

Table 3. Distribution of households by deciles of total income per person, in 2006

	Decile 1	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10
Monthly income per person	119,000	147,000	200,000	252,000	307,500	367,500	441,000	541,000	709,500	821,000
persons (percent)	0,133000	0,109000	0,102000	0,101000	0,098000	0,094000	0,096000	0,094000	0,091000	0,082000
m.f.1	1,000000	0,432041	0,175174	0,061647	0,018317	0,004347	0,000638	0,000042	0,000000	0,000000
m.f.2	1,000000	0,747149	0,504880	0,298304	0,150169	0,061520	0,016727	0,002179	0,000033	0,000000
m.f.3	1,000000	0,284837	0,072857	0,014873	0,002319	0,000253	0,000013	0,000000	0,000000	0,000000

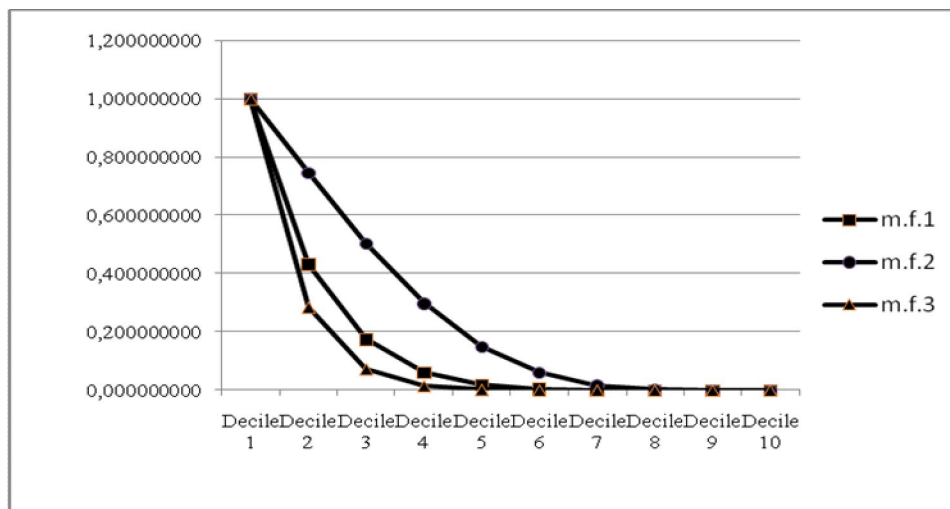
Income intervals are expressed in the prices of January 2006.

**Table 4.** Distribution of in households by deciles of total income per person, in 2003

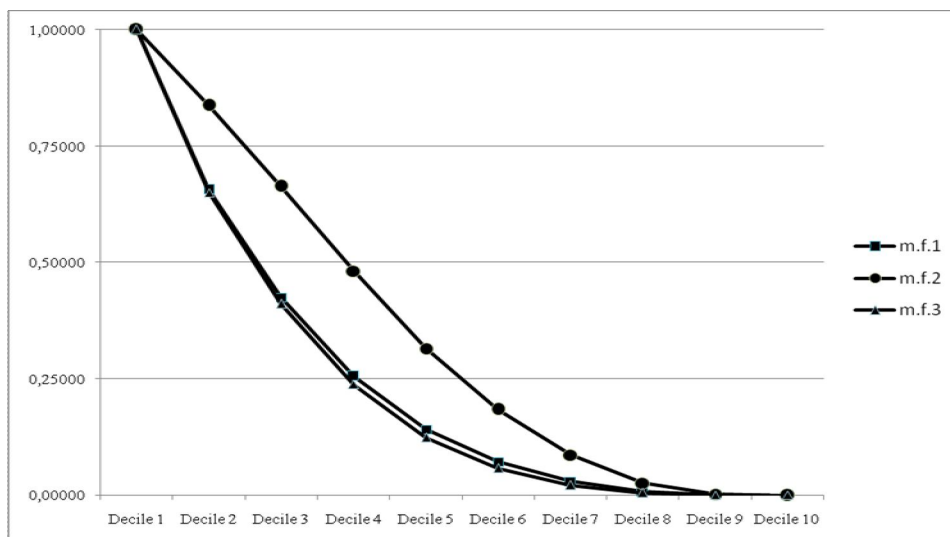
	Decile 1	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10
Monthly income per person	603,500	763,088	1063,545	1354,649	1672,503	2004,930	2369,519	2861,490	3733,929	4314,208
persons (percent)	0,129000	0,114000	0,102000	0,102000	0,099000	0,092000	0,092000	0,094000	0,093000	0,083000
m.f.1	1,000000	0,656498	0,425275	0,255930	0,141616	0,071791	0,029788	0,008251	0,000865	0,000000
m.f.2	1,000000	0,837693	0,663364	0,481686	0,315353	0,184409	0,086836	0,026023	0,002076	0,000000
m.f.3	1,000000	0,650222	0,411435	0,237870	0,123920	0,057834	0,021225	0,004777	0,000312	0,000000

*Income intervals are expressed in the prices of January 2006.*

The graphics of the membership functions for individuals are illustrated in Figure 4 and Figure 5.



**Fig. 4.** Graphic of the membership functions for individuals m.f.1, m.f.2, and m.f.3 for 2006



**Fig. 5.** Graphic of the membership functions for individuals m.f.1, m.f.2, and m.f.3 for 2003

We may observe that also for this case, for both years 2003 and 2006, the graphic of m.f.1 is situated between the curves of m.f.2 and m.f.3. Moreover, for year 2003, the values computed for m.f.2 and m.f.3 are very similar.

## Conclusion

Remarks 1 and 2 lead us to the conclusion that the degree of poverty is a fuzzy  $\prec$ -measure, and, moreover, a fuzzy (monotone) measure with respect to the specified set inclusion preorder.

We consider the third variant of fuzzy membership function to be the most appropriate for poverty measurement, as it takes into account both the share of individuals less poor than the people concerned and also the share of the total equivalised income received by all individuals less poor than the people concerned.

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## Construcția funcțiilor de apartenență pentru măsurile fuzzy utilizate în măsurarea sărăciei

### Rezumat

Articolul prezintă o introducere în teoria mulțimilor vagi, care, în ultima perioadă, este tot mai mult utilizată în studiile de măsurare a sărăciei sub aspect multi-dimensional. Abordarea multi-dimensională în măsurarea și analiza sărăciei implică atât aspecte monetare cum ar fi incidența și intensitatea sărăciei generată de un venit scăzut, cât și aspecte ne-monetare precum accesul scăzut la alte resurse, facilități, și chiar atributele individuale care pot determina sau influența stilul de viață.

Considerăm că este mult mai potrivit să privim sărăcia ca pe ca un atribut vag, difuz, ce exprimă diferitele grade de manifestare a fenomenului, atribuind expresii cantitative intensității de manifestare a fenomenului, pentru indivizi diferiți și referitor la diferite dimensiuni ale sărăciei, și nu ca pe un atribut ce caracterizează un individ strict sub aspectul prezenței sau absenței sale. Articolul privește numai dimensiunea monetară a sărăciei și se concentrează pe aspectele metodologice legate de construcția funcțiilor de apartenență care sunt ulterior extinse la măsuri monetare fuzzy. În ultima parte a articolului sunt construite măsurile fuzzy de măsurare a sărăciei pentru gospodăriile distribuite pe decile, pentru anii 2003 și 2006 în România.