

Comparison of EIA, IEA and OPEC Projections for Global Oil Demand and Non-OPEC Supply with Grey Theory

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Abstract

The Grey Theory analysis the systems with incomplete and poor data; moreover the Markov Model represents the volatility law and optimizes the result. We analyzed the EIA, IEA and OPEC Global Oil Demand and Non OPEC Supply projections with the GM (1, 1) and compare them with the actual values. We improve our model forecasting for EIA Global Oil Demand and IEA, Non OPEC Supply by the Grey Markov model.

Keywords: *Grey Markov Model, Global Oil Demand, Non-OPEC Supply, EIA, IEA, OPEC*

JEL Classification: *C530*

Introduction

Julong Deng in 1982 established the grey system theory which studies the problems containing poor information and small samples. The traditional statistical analysis requires large amounts of data but the grey method can be used only by inadequate data. The data with any distribution can be used in grey systems but in probabilistic and statistical methods only the data with typical distribution are required. The grey theory treats uncertain systems and provides useful information from what is available. This theory is broadly applied in fields such as systems modeling, analysis, data processing, prediction, control and decision-making.

Grey predictions can be categorized as system, interval, serial, disaster, seasonal disaster and stock market like predictions. Chaos, complexity, order and indulgence, tolerance, security corresponding are the property and attitude of black, grey and white systems, respectively (Deng, 1982).GM(1,1) model is the most frequent model in grey forecasting in which the raw

data sequence must be altered to exponential rule; moreover, a grey markov model is a probabilistic method which organized the relative deviation between the GM(1,1) value and actual, estimates the transition probability matrix and formulates the development trends in the future. This model has no memory.

Many researchers have applied the grey models in many fields: EL-Fonly et al (2006) applied the GM(1,1) model to forecast the wind speed and power. Singgih and Pamungkas(2009) applied different kinds of grey model to increase Juand international airport customer satisfaction. Chi et al (1999) predict the future Taiwan stock index by the grey forecasting model and filtered the most influential indices by the grey relationship analysis. C.Li (2006) applied a grey markov model based on parameter fits to build a model for Shanghai stock market price. Wei and Jinfu (2009) forecasted the fluctuations of passenger traffic by the grey markov model. Xin et al (2004) applied the GM(1, 1) with the function transform method to forecast the energy consuming trend. He and Huang(2005) forecasted the electric power requirement in China by a grey markov model. Zhi and Yi(2009) used the grey markov model for forecasting the wear trend of diesel engine. Jou et al (2008) studied a grey system theory for travel time prediction and used markov bias correction mechanism for bias correction. Jiang et al (2009) predicted the volume of water used by a grey markov model. Dang et al (2012) forecasted the maximum water level at hydrological stations by a grey markov model. Li Juan(2012) reflected the change in the Jingdezhen ceramic industrial output by the improved grey model . Zhang Y. et al (2009) forecasted the Dalian energy consumption by a grey model with partial least square regression. Ma H. and Zhang D. (2009) forecasted the coal production and consumption in China by the grey markov model.

There are different organizations which produce energy outlooks. Energy forecasting influences the development of energy policies. Modeling and evaluating the accuracy of past projections leads to improvements in projections over time and presents advantageous information for identifying in current projections. This study compares energy forecasts from energy information administration (EIA), International Energy Agency (IEA) and Organization of the Petroleum Exporting Countries (OPEC) and introduces a method to model the projections and show the differences amongst them. Some researchers have analyzed different energy scenarios: Silbergliitt R. et al (2003) compare different energy scenarios quantitatively to United States energy consumption efficiency and carbon content. Molinari T. (2012) analyzed the EIA, IEA, BP and Exxon Mobil outlooks for United States and North America. H. Douglas Lightfoot (2007) analyzed the calculation methods various definitions and metrics of primary energy by different organizations.

In this paper first we describe the fundamental definitions of grey theory and the mathematical model. Second we compare the EIA,IEA and OPEC projections for global demand and non OPEC supply by the GM (1,1). Finally we improve our model precision in two instances by the grey markov.

The Mathematical Methods

The procedure of our mathematical prediction model can be summarized as follows:

Suppose that $X^{(0)}(k) = \{X^{(0)}(1), X^{(0)}(2), \dots, X^{(0)}(n)\}$ is the original data. A new sequence $X^{(1)}$ is set up through accumulated generating as follows:

$$X^{(1)}(k) = (X^{(1)}(1), X^{(1)}(2), \dots, X^{(1)}(n)) \tag{1}$$

Where

$$X^{(1)}(k) = \sum_{i=1}^k X^{(0)}(i) , k = 1, 2, \dots, n.$$

The Grey prediction GM(1,1) model can be expressed by one variable and first order differential equation

$$\frac{dX^{(1)}(k)}{dk} + aX^{(1)}(k) = b \tag{2}$$

In reality the parameters a and b are not calculated directly from equation(2).Hence the solution of (2) can be gained by using the least square method; Moreover according to Eq. (2), the solution of $\hat{X}^{(1)}(k)$ at time k is :

$$\hat{X}^{(1)}(k) = \left(X^{(0)}(1) - \frac{b}{a} \right) e^{-ak} + \frac{b}{a} \tag{3}$$

Where

$$\begin{pmatrix} a \\ b \end{pmatrix} = (B^T B)^{-1} B^T Y_n \tag{4}$$

And

$$B = \begin{pmatrix} -\frac{1}{2}(X^{(1)}(1) + X^{(1)}(2)) & 1 \\ -\frac{1}{2}(X^{(1)}(2) + X^{(1)}(3)) & 1 \\ \vdots & \vdots \\ -\frac{1}{2}(X^{(1)}(n-1) + X^{(1)}(n)) & 1 \end{pmatrix} \tag{5}$$

$$Y_n = \begin{bmatrix} X^{(0)}(2) \\ X^{(0)}(3) \\ \vdots \\ X^{(0)}(n) \end{bmatrix} \tag{6}$$

By inverse accumulative generating operation, the predicted equation is:

$$\hat{X}^{(0)}(k) = \left(X^{(0)}(1) - \frac{b}{a} \right) (1 - e^{-ak}) e^{-ak} \tag{7}$$

Suppose

$$\hat{Y}(k) = \hat{X}^{(0)}(k+1) \tag{8}$$

Since \hat{Y} is a Markov Chain , we can divide it into several zones which are parallel to the regulation curve in accordance with particular circumstance and any zone H_i can be stated as :

$$H_i = [\hat{H}_{1i}, \hat{H}_{2i}] \quad i=1, 2, 3, \dots, n \tag{9}$$

Were

$$\hat{H}_{1i} = \hat{X}^{(0)}(k+1) + A_i \quad i=1, 2, 3, \dots, n \tag{10}$$

$$\hat{H}_{2i} = \hat{X}^{(0)}(k+1) + B_i \quad i=1, 2, 3, \dots, n \tag{11}$$

Here A_i, B_i are constant, which can be obtained by the difference between the raw data and forecasting regulation curve. We defined the borderlines of the zones above the regulation curve as upper borderlines and the ones under as lower borderlines. The upper and lower borderlines are assumed as $\hat{X}^{(0)}(k+1) + A$ and $\hat{X}^{(0)}(k+1) - B$, respectively. A and B are obtained by using the least square method as

$$A = (\sum_H X^{(0)}(H+1) - \sum_H \hat{X}^{(0)}(H+1)) / p \tag{12}$$

$$B = (\sum_L X^{(Q)}(L+1) - \sum_L \hat{X}^{(Q)}(L+1))/q \quad (13)$$

Where $X^{(Q)}(H+1)$ is the observed data above the regulation curve, p is the number of these data, $X^{(Q)}(L+1)$ denotes the observed data below the regulation curve and q is the number of these lower data. Let $\hat{X}^{(Q)}(k+1)+C$ and $\hat{X}^{(Q)}(k+1)-D$ as the top and bottom borderlines, respectively where

$$C = \max \{ X^{(Q)}(k+1) - \hat{X}^{(Q)}(k+1) \} \quad (14)$$

$$D = \max \{ \hat{X}^{(Q)}(k+1) - X^{(Q)}(k+1) \} \quad (15)$$

By getting $A, B, C,$ and $D,$ we have four zones as follows:

$$\begin{aligned} H_1 &= [\hat{X}^{(Q)}(k+1)+A, \hat{X}^{(Q)}(k+1)+C] \\ H_2 &= [\hat{X}^{(Q)}(k+1), \hat{X}^{(Q)}(k+1)+A] \\ H_3 &= [\hat{X}^{(Q)}(k+1)-B, \hat{X}^{(Q)}(k+1)] \\ H_4 &= [\hat{X}^{(Q)}(k+1)-D, \hat{X}^{(Q)}(k+1)-B] \end{aligned} \quad (16)$$

More subzones can be divided in each zone above with the same method.

If $M_{ij}(m)$ is the data number of raw series which transfer m step from H_i to H_j and M_i is the number of data that is in the zone H_i , then we call

$$p_{ij} = \frac{M_{ij}(m)}{M_i} \quad i, j = 1, 2, 3, \dots, n(17)$$

the m th step transition probability. The transition matrix $P(m)$ is as follow:

$$P(m) = \begin{bmatrix} p_{11}(m) & p_{12}(m) & \dots & p_{1n}(m) \\ p_{21}(m) & p_{22}(m) & \dots & p_{2n}(m) \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1}(m) & p_{n2}(m) & \dots & p_{nn}(m) \end{bmatrix} \quad (18)$$

$P(m)$ reflects the transition regulation between different states and is the foundation of the forecast model of Grey Markov. We can predict the future trend of the systems by studying the stochastic transition matrix $P(m)$. If $P(1)$ has more than two lines whose probability values are same alike or close to other and it is difficult to decide the next direction of the system with certain, it is needed to study and check the matrix $P(2)$ or $P(m)(m \geq 3)$. At the same time, it can be decided the transition of the system by checking $P(1)$ or $P(m)(m \geq 2)$. At last the eventual forecast value can be obtained as

$$\hat{Y}'(k) = \frac{1}{2} (\hat{H}_{11} + \hat{H}_{21}) \quad (19)$$

Applying (10), (11) and since the forecast is most probably in zone H_1 , then $\hat{Y}'(k)$ can be expressed as

$$\hat{Y}'(k) = \hat{X}^{(Q)}(k+1) + \frac{1}{2} (A_1 + B_1) \quad (20)$$

Applications

We will model the IEA, EIA and OPEC forecasting for Global Oil Demand and Non OPEC Supply by the grey system theory. The data are listed in Table 1.

Table 1. Latest estimates of EIA, IEA and OPEC for world oil demand and non OPEC supply (mb/d)

Date	Global Oil Demand (mb/d)			Non OPEC Oil Supply(mb/d)		
	EIA	IEA	OPEC	EIA	IEA	OPEC
Q ₁ 2011	87.48	87.59	86.11	51.49	53.3	52.31
Q ₂ 2011	86.95	86.95	85.69	51.73	52.8	52.19
Q ₃ 2011	87.44	87.44	86.74	50.49	52.6	52.27
Q ₄ 2011	87.55	87.55	87.67	50.66	53.1	52.91
Q ₁ 2012	88.40	88.40	87.93	52.36	53.4	53.23
Q ₂ 2012	88.50	88.50	87.69	52.24	52.9	52.64
Q ₃ 2012	89.96	89.96	89.61	52.33	53.0	52.67
Q ₄ 2012	89.47	89.47	89.99	53.18	53.6	53.46
Q ₁ 2013	89.56	89.56	88.87	52.92	53.5	53.52
Q ₂ 2013	89.22	89.22	88.34	53.56	53.8	53.53
Q ₃ 2013	90.69	90.69	90.34	54.21	53.9	53.87
Q ₄ 2013	90.53	90.53	90.77	54.34	54.5	54.61

Source: *www.reuters.com*

The coefficients of GM(1,1) for these data are listed in Table 2.

Table 2. The estimated coefficients of GM(1,1) for EIA, IEA and OPEC for global oil demand and non OPEC supply (mb/d)

Coefficients	Global Oil Demand(mb/d)			Non OPEC Oil Supply(mb/d)		
	EIA	IEA	OPEC	EIA	IEA	OPEC
a	-0.00415	-0.00366	-0.00461	-0.00700	-0.00239	-0.00317
b	86.58299	87.37158	85.57275	50.16683	52.44840	52.02794

Source: made by the authors

The GM(1,1) for the EIA, IEA and OPEC fore castings and the actual value are obtained and plotted (Figures 1-4).

(Global Oil Demand) EIA: $\hat{X}^{(0)}(K+1) = 86.765700e^{0.00414732k}$

IEA: $\hat{X}^{(0)}(K+1) = 87.53187834e^{0.00366k}$

OPEC: $\hat{X}^{(0)}(K+1) = 86.067369e^{0.0046114k}$

(Non OPEC Oil Supply) EIA: $\hat{X}^{(0)}(K+1) = 50.35082975e^{0.0070012k}$

IEA: $\hat{X}^{(0)}(K+1) = 52.5629749e^{0.00239103k}$

OPEC: $\hat{X}^{(0)}(K+1) = 52.11105139e^{0.00316721k}$

From the figures we understand that in short term, the IEA projections for global demand are the most realistic forecastings. We conclude that after six years IEA projections will be in accordance with the actual values. It is inferred that approximately ten to thirty years, the EIA forecastings for global demand will be the most adjacent projections. In long term, the IEA projections are the most reliable and realistic for global demand.

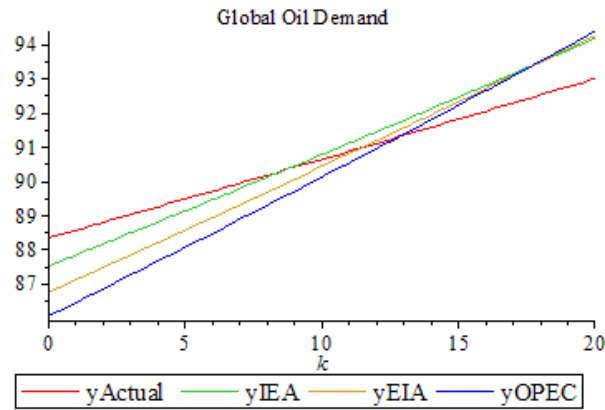


Fig. 1. Modeling the EIA, IEA and OPEC Global Oil Demand projections for short time(mb/d)
Source: made by the authors

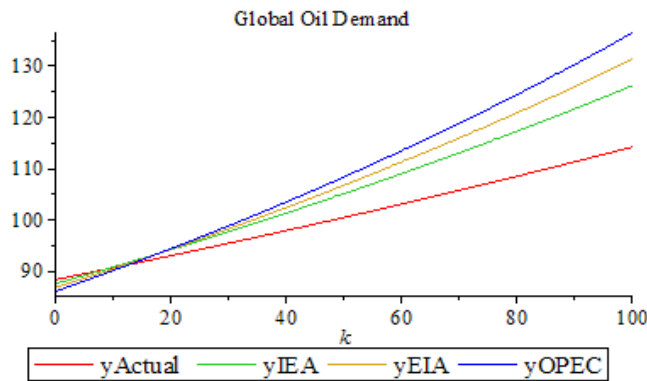


Fig. 2. Modeling the EIA, IEA and OPEC global oil demand for long time (mb/d)
Source: made by the authors

In short and long term , the IEA projections are more near to the actual trend than EIA and OPEC projections for non OPEC supply are in accordance with the actual trend for some years before the tenth year of prediction:

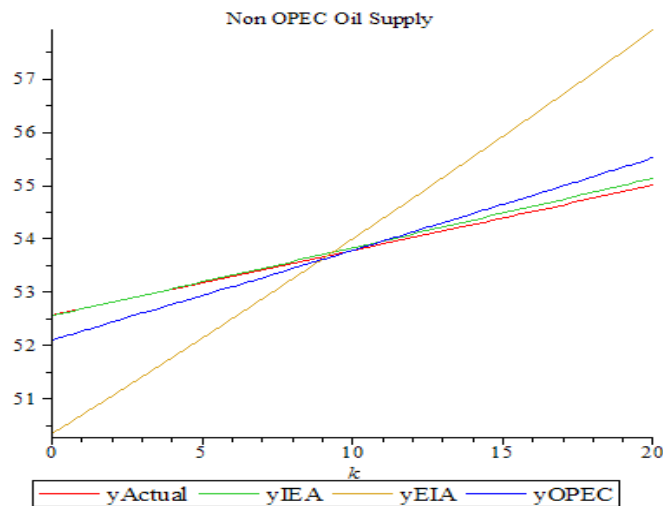


Fig. 3. Modeling the EIA, IEA and OPEC non OPEC oil supply for short term (mb/d)
Source: made by the authors

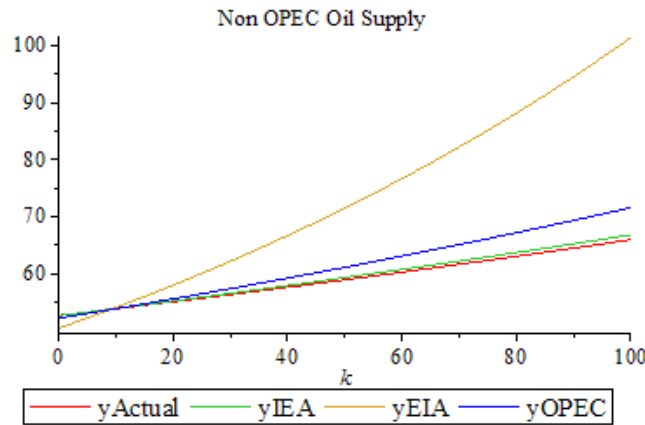


Fig. 4. Modeling the EIA, IEA and OPEC non OPEC oil supply for long term(mb/d)

Source: made by the authors

The high precisions of forecastings (table 3) show that the GM(1,1) is a reliable model for assessing the EIA, IEA and OPEC projections.

Table 3. The result of GM (1,1) for EIA, IEA and OPEC projections(mb/d)

	EIA			IEA			OPEC		
	Forecasted value	Actual value	Precision	Forecasted value	Actual value	Precision	Forecasted value	Actual value	Precision
Global Demand	90.82	90.53	99.68%	91.12	91.5	99.58%	90.55	90.77	99.76%
Non OPEC Supply	54.38	54.37	99.98%	53.96	54.5	99.01%	53.96	54.61	98.81%

Source: made by the authors

We can predict the future projections of these organizations by the estimated equations and improve their forecasting precisions by modifying the grey models. Now we improve the EIA global oil demand and IEA Non OPEC Supply projections by the Grey Markov model. The results on grey markov coefficients for EIA global oil demand are as follow:

$$A = 0.46, B = 0.263, C = 1.008, D = 0.85$$

The zones concerning the markov model states are obtained as:

$$\begin{aligned}
 H_1 &= [\hat{X}^{(0)}(k+1) + 0.46, \hat{X}^{(0)}(k+1) + 1.008] \\
 H_2 &= [\hat{X}^{(0)}(k+1), \hat{X}^{(0)}(k+1) + 0.46] \\
 H_3 &= [\hat{X}^{(0)}(k+1) - 0.263, \hat{X}^{(0)}(k+1)] \\
 H_4 &= [\hat{X}^{(0)}(k+1) - 0.85, \hat{X}^{(0)}(k+1) - 0.263],
 \end{aligned}$$

which plotted in Figure 5.

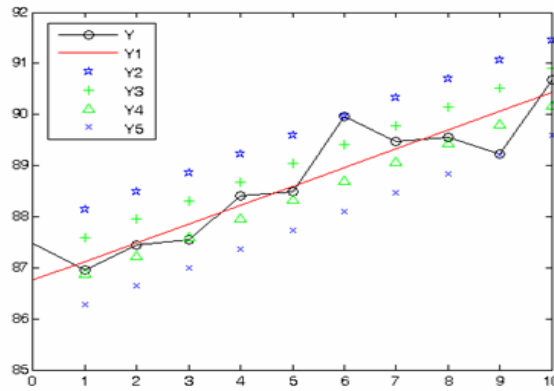


Fig. 5. The predicting curve for EIA Global Demand(mb/d)

Source: made by the authors

In this step by calculating the transition probabilities, we concluded that the Q_3 2013 is in region H_2 and therefore the maximum probability of the second row is p_{23} .

$$P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0.33 & 0.67 & 0 \\ 0 & 0.25 & 0.25 & 0.5 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

As a result the next transition is from H_2 to H_3 . Finally we

obtain the Q_4 2013 as:

$$\hat{Y}^{\hat{}}(11) = \hat{X}^{(0)}(12) - \frac{1}{2}B = 90.69$$

By calculating the precision for markov model we conclude that the grey markov model gives the more accurate result than GM (1, 1) (Table 4).

Table 4. The result of grey markov model for EIA Global Oil Demand(mb/d)

	Forecasted value (Q ₄ 2013)	Actual value	Precision
EIA projection for Global Oil Demand	90.69	90.53	99.82%

Source: made by the authors

The results of grey markov coefficients for IEA Non OPEC Oil Supply are as follow:

$$A = 0.175, B = 0.2, C = 0.7303, D = 0.3225$$

The zones concerning the markov model states are obtained as:

$$H_1 = [\hat{X}^{(0)}(k+1) + 0.175, \hat{X}^{(0)}(k+1) + 0.73703]$$

$$H_2 = [\hat{X}^{(0)}(k+1), \hat{X}^{(0)}(k+1) + 0.175]$$

$$H_3 = [\hat{X}^{(0)}(k+1) - 0.2, \hat{X}^{(0)}(k+1)]$$

$$H_4 = [\hat{X}^{(0)}(k+1) - 0.3225, \hat{X}^{(0)}(k+1) - 0.2]$$

The four regions and their borderlines are plotted in Figure 6.

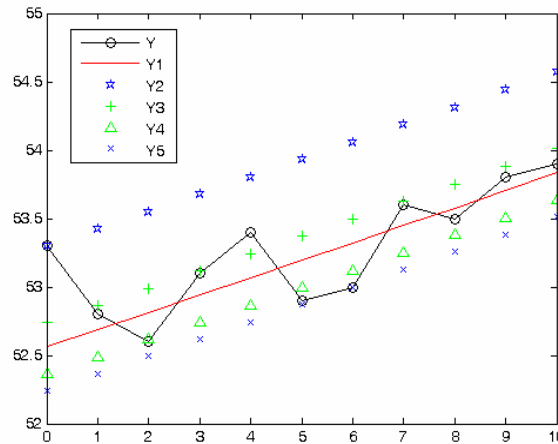


Fig. 6. The predicting curve for IEA non OPEC supply(mb/d)

Source: made by the authors

The one step transition probability matrix is computed as $P = \begin{bmatrix} 0 & 0.5 & 0 & 0.5 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0 & 1 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 \end{bmatrix}$

from figure 6. We conclude that Q_3 2013 is in H_2 ; moreover the probabilities in the second row all are equal and therefore we must calculate the second step transition probability matrix to obtain the next most probable state.

$$P^{(2)} = \begin{bmatrix} 0.125 & 0.375 & 0.125 & 0.375 \\ 0.063 & 0.563 & 0.063 & 0.3125 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.125 & 0.375 & 0.125 & 0.375 \end{bmatrix}$$

Finally we conclude that the next state (Q_4 2013) will be in H_1 and therefore

$$\hat{Y}^{(11)} = \hat{X}^{(0)}(12) + \frac{1}{2}A = 54.05$$

The forecasting precision of grey markov model is more accurate than GM (1, 1) (Table 5).

Table 5. The result of grey markov model for IEA non OPEC supply(mb/d)

	Forecasted value	Actual value	Precision
IEA projections for non OPEC oil supply	54.05	54.5	99.17%

Source: made by the authors

Conclusion

Grey model is used in small samples with less uncertainty in a short period and markov model is applied to dynamic process. GM(1,1) is commonly applied to predict fluctuation sequence. We modeled the EIA, IEA and OPEC world oil demand and non OPEC supply with GM(1,1). Comparing their model with the actual value, in the case of global oil demand it is concluded that IEA projections are the most actual results for now up to tenth year and in the long term they give the most reliable predictions. In the period from next ten to thirty years the EIA forecastings are more accurate.

In the case of Non OPEC Supply, the IEA projections are the closest results to the actual values. However for some years before the tenth year the OPEC forecastings are the most reliable. The

precision of the forecasting was improved for EIA Global Oil Demand and IEA Non OPEC Supply by the Grey Markov model.

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