The Prediction of Monthly Inflation Rate in Romania¹

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Abstract

Predictions for inflation rate are constructed for underlying the decisional process at macroeconomic level. The central banks also build forecast intervals for inflation rate to reflect the degree of uncertainty of their point predictions. The purpose of constructing forecast intervals is related to the improvement of macroeconomic policies and of decisional process. The utility of this demarche is higher for the National Bank of Romania that proposes inflation rate prediction intervals in the context of indicator targeting. In this paper, prediction intervals are constructed for monthly inflation rate in Romania using various methods: the bootstrap technique (percentile prediction intervals and percentile-t prediction intervals) and historical errors method based on root mean square error. The latter method started from a moving average process of the inflation rate, the static and dynamic forecasts not having a high degree of accuracy, but being unbiased. For the monthly forecasts over March 2014-June 2014, the bootstrapped forecast intervals based on a simple regression model where the unemployment rate is the exogenous variable are less plausible than the intervals based on historical root mean square error. However, in all the cases the inflation has a tendency to increase in time. This research could be developed in order to construct a fan chart for inflation rate in Romania, this graph being based on forecast intervals.

Keywords: prediction intervals, bootstrap method, historical errors method, inflation rate

JEL Classification: E21, E27,C51, C53

Introduction

The construction of prediction intervals became a necessary demarche to reflect the degree of uncertainty that characterizes the forecasting process. For a certain probability of guaranteeing the results, some intervals are proposed for the future values of inflation. This approach is necessary in underlying the decisional process at macroeconomic level and in establishing the best policies. The anticipation of high values for inflation rate will determine the taking of measures to diminish the negative consequences of this process evolution. In Romania the National Bank constructs uncertainty intervals for inflation by keeping constant the value of a forecast accuracy indicator. The targeting inflation process requires a careful monitoring of the future evolution of the inflation rate in Romania. A simple point forecast is not enough,

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providing few chances to register that inflation rate value. The prediction interval by its construction admits the uncertainty, giving a zone where the inflation could be placed for high chances. There are several methods for constructing forecast intervals, the bootstrap technique being used lately to predict the economic indicator because of the lack of restrictions supposed by the other methods that assume a classical distribution of the error.

The main objective of this article is to construct prediction intervals for inflation rate in Romania using the bootstrap method, but also the historical error method based on the past values of an accuracy indicator. The article is structured as follows. After the brief presentation of the literature, the article discusses the methodological background and the research with the corresponding results for predicting inflation rate in Romania. The main results indicate that historical error method provided more plausible forecast intervals for the monthly inflation rate in Romania, even if the intervals show a tendency of increase form a period to another. The last section draws the main conclusions.

The bootstrap technique is a method of generating sample distribution that can be used when the type of repartition is not known (Efron and Tibshirani, 1993). The bootstrap technique supposes the replacement of elements from the sample, each observation having the same probability to be selected. The means of all generated samples are registered. A larger population normally distributed is chosen and its parameters are estimated and the repartition of sample means are determined. The variables used in this study are the inflation rate and the unemployment rate. The inflation rate measures the change in the index of consumer prices for the goods and services from consumer basket. The unemployment rate shows the number of unemployed people in the active population. The variables values are published by the National Institute of Statistics from Romania. In the crisis context, the inflation rate and the unemployment rate have increased, but in the first months of 2014 the indicators have registered irregular evolutions with sudden decreases and increases.

Efron and Tibshirani showed that the bootstrap technique is used to estimate the sampling distribution of a statistic, the repartition not being known, by repeating the re-sampling of the original data set (Efron and Tibshirani, 1993). McCullough applied bootstrap method to estimate forecast intervals for an AR(p) model. Box-Jenkins prediction intervals were compared to those based on naïve bootstrap and a bias-correction (BC) bootstrap (McCullough, 1994). The differences between the three forecasting methods are quite large.

Miguel and Olave used a bootstrap technique for building forecast intervals for an ARMA model when the errors follow an autoregressive conditional heteroscedastic process (Miguel and Olave, 1999). The validity of this technique was demonstrated using simulations.

Reeves built forecast intervals for ARCH models using parametric and non-parametric bootstrap methods, these techniques improving the accuracy of classical asymptotic prediction intervals (Reeves, 2000).

MacKinnon considers bootstrap technique as a good alternative to the classical methods used to make estimations or forecasts (MacKinnon, 2002). When an AR model is used, the bootstrap method supposes the generation of many pseudo-data based on re-sampled residual and on the estimated parameters of the model.

Gospodinov used the grid bootstrap method proposed by Hansen to determine forecasts with unbiased median in the cases of the processes with a high degree of persistence (Gospodinov, 2002). According to Guan the bootstrap techniques gives good results but it is computationally intensive (Guan, 2003).

Alonso, Pena and Romo used a sieve bootstrap method for building nonparametric forecast intervals for a general group of linear models (Alonso, Pena and Romo, 2003). This technique allows consistent estimators of the conditional distribution of the next values.

Methodology

We start from the multiple linear regression model:

$$Y = X\beta + u \tag{1}$$

Y- vector (dimension: nx1)

X- matrix (dimension: nxp)

- vector of coefficients (dimension: px1)

u- vector of random errors (dimension: nx1)

 $\hat{\mathbf{u}}$ - residuals $(\hat{\mathbf{u}} = \mathbf{Y} - \mathbf{X}\hat{\mathbf{\beta}})$

 $\hat{\beta}$ - estimator $(\hat{\beta} = (X^T X)^{-1} X^T Y)$

The bootstrap model form is:

$$Y^* = X\hat{\beta} + u^* \tag{2}$$

Y*- vector (dimension: nx1)

X*- matrix (dimension: nxp)

 $\hat{\beta}$ - estimator $(\hat{\beta} = (X^T X)^{-1} X^T Y)$

u - random term got from the residual of the initial regression

At each iteration b, a sample is extracted from the bootstrap regression model: The random element from the theoretical bootstrap model is based on the following transformed residuals:

$$\widetilde{u}_i = \frac{\widehat{u}_i}{\sqrt{1 - h_i}} - \frac{1}{n} \sum_{s=1}^n \frac{\widehat{u}_s}{\sqrt{1 - h_s}}$$
(3)

The theoretical bootstrap model is determined as:

$$y_i^*(b) = X_i \hat{\beta} + \tilde{u}_i^*(b) \tag{4}$$

i=1,2,...,n

 $\overline{u_i}(b)$ - resampled from $\overline{u_i}$

Starting from the random variable $z_j = \frac{\beta_j - \beta_j}{s(\beta_j)}$, the standard confidence interval of β_j is based on the assumption that z_j follows a Student reparation with n-p degrees of freedom. A (1-2 α) confidence interval is determined as:

$$[\hat{\beta}_j - s(\hat{\beta}_j)t_{(a-\alpha),n-p}; \hat{\beta}_j - s(\hat{\beta}_j)t_{(\alpha),n-p}]$$
(5)

A $(1-2^{\alpha})$ percentile confidence interval is determined as:

$$\left[\widehat{\beta}_{j}^{*}(\alpha B)_{i}\widehat{\beta}_{j}^{*}((1-\alpha)B)\right] \tag{6}$$

The percentile-t bootstrap technique supposes the estimation of the distribution function of \mathbb{Z}_j using the data. A bootstrap table is built, \mathbb{Z}_j^* being computed as:

$$\mathbf{z}_{i}^{*} = \frac{\widehat{\beta}_{j}^{*} - \widehat{\beta}_{j}}{s^{*}(\widehat{\beta}_{i}^{*})} \tag{7}$$

The percentile-t confidence interval of β_i is:

$$\left[\beta_{j} - s\left(\beta_{j}\right)\xi^{(1-\alpha)};\beta_{j} - s\left(\beta_{j}\right)\xi^{(\alpha)}\right] \tag{8}$$

The bootstrap technique could be used to build forecast intervals on regression models that have fixed repressors and unknown values for them. McCullough (1996) proposed bootstrap prediction intervals on models that have stochastic repressors. For the f-th observation of independent variable X, the forecast is computed using the regression model, where Y is the dependent variable: $\hat{Y}_f = X_f \hat{\beta}$. The errors follow a normal distribution and the confidence interval (1-2 α), the standard forecast interval has the following form:

$$[\hat{y}_f = s_f \cdot t_{(1-\alpha),n-p}; \hat{y}_f + s_f \cdot t_{(1-\alpha),n-p}]$$
(9)

The error distribution of the forecast should be analyzed in order to apply bootstrap technique in constructing forecast intervals. The bootstrap prediction intervals are computed by bootstrapping the residuals. There are two techniques of building bootstrap prediction intervals: the percentile method and the percentile-t one.

In order to construct the percentile intervals the bootstrap approximation of the forecast error repartition is used. A forecast interval for y_f is built using $e_f = \hat{y}_f = y_f$. The following model is used to obtain the bootstrap replication of the next value \hat{y}_f^* :

$$y_f = X_f \hat{\beta} + \tilde{u}_f \tag{10}$$

The prediction error $\mathfrak{A}_{f}^{\bullet}$ is computed from a kind of retrieval in the empirical repartition of the transformed residuals. For every bootstrap replication from the B replication, the bootstrap estimator is computed. The forecast and the bootstrap forecast error are calculated as:

$$\widehat{y}_f^*(b) = X_f \widehat{\beta}^*(b) \tag{11}$$

$$e_f^*(b) = \hat{y}_f^*(b) - y_f^*(b)$$
 (12)

The bootstrap forecast error is rewritten as:

$$e_f^* = \hat{y}_f^* - \hat{y}_f - \tilde{u}_f^* \tag{13}$$

The empirical repartition of \mathcal{E}_f^* denoted by \mathcal{G}^* are given by the B replications of the forecast error. The percentiles are used to build the bootstrap forecast intervals $(\mathcal{G}^{-1}(1-\alpha)and\mathcal{G}^{-1}(\alpha))$. The percentile forecast interval is determined as:

$$[\hat{y}_f - G^{\tau-1}(1-\alpha); \hat{y}_f - G^{\tau-1}(\alpha)]$$
 (14)

For constructing the percentile-t forecast interval for each bootstrap sample the estimator of the standard deviation (\mathbb{S}^*) is determined:

$$s_{\downarrow}f^{\uparrow} := s^{\uparrow} * \cdot \sqrt{(1 + h_{\downarrow}f)}$$
 (15)

$$\mathbf{h}_f = X_f (X^T X)^{-1} X_f^T \tag{16}$$

The statistic ** is computed, its bootstrap distribution defining the percentile-t bootstrap forecast interval:

$$Z_f^* = \frac{e_f^*}{s_f^*} = \frac{\hat{y}_f^* - \hat{y}_f - \hat{u}_f^*}{s_f^*}$$
(17)

The percentile-t forecast interval is computed as:

$$\hat{y}_f - s_f \cdot z_{f(1-\alpha)}^* \hat{y}_f - s_f \cdot z_{f(\alpha)}^*$$
(18)

For a moving average process in describing the evolution of our indicator, the prediction at a future time "n+h" has the following form:

$$\inf_{i = h} \sum_{j=0}^{h-1} c_j e p s_{n+h-j} + \sum_{j=h}^{\infty} c_j e p s_{n+h-j}$$
(19)

- the coefficient

j- the index of time

The best forecast (f) is in this case:

$$f_{n,h} = \sum_{j=h}^{\infty} c_j eps_{n+h-j}$$
(20)

In our case, for one-step-ahead predictions, h equals 1 and the prediction is $f_{n,1} = c_1 eps_n$.

The forecast error is given by:

$$e_{n,h} = inf_{n+h} - f_{n,h} = \sum_{j=0}^{h-1} c_j eps_{n+h-j}$$
(21)

The mean of forecast errors is considered to be null. The errors' variance is:

$$var(e_{n,k}) = \sigma_{eps}^2 \sum_{j=0}^{h-1} c_j^2$$

In our particular case, the variance is: eps_n^2

Considering the hypothesis that the errors distribution is a normal one, the forecast interval is

determined as: $f_{n,h} \pm 1.96$ var $(e_{n,h})$. In our case, the forecast interval has the following form: $c_1 \cdot eps_n \pm 1.96 \cdot eps_n$, that becomes $eps_n \cdot (c_1 \pm 1.96)$.

The forecast interval based on historical errors method assumes the errors' normal distribution of null average and a standard deviation given by the root mean squared error of the historical errors. For a level of significance of α , the prediction intervals have the following form:

$$\hat{y}_f = RMSE(k) \cdot z_{\left(\frac{\alpha}{2}\right)^2} \hat{y}_f + RMSE(k) \cdot z_{\left(\frac{\alpha}{2}\right)}$$
(22)

 \hat{Y}_{f} = the point forecast of Y made at time t for period (t+k)

 \P - quintile $\alpha/2$ of standard normal distribution

Prediction Intervals for Inflation Rate in Romania

The variables used in this study are the unemployment rate and the inflation rate registered for the Romanian economy. The monthly data were used, the dependent variable being the inflation rate based on index of consumer prices in comparable prices. The data are registered for the period from January 2004 to February 2014. The predictions are made on the horizon March 2014-June 2014. The inflation rate measures the change in the index of consumer prices for the goods and services from consumer basket. The unemployment rate shows the number of unemployed people in the active population. The variables values are published by the National Institute of Statistics from Romania. In the crisis context, the inflation rate and the

unemployment rate have increased, but in the first months of 2014 the indicators have registered irregular evolutions with sudden decreases and increases.

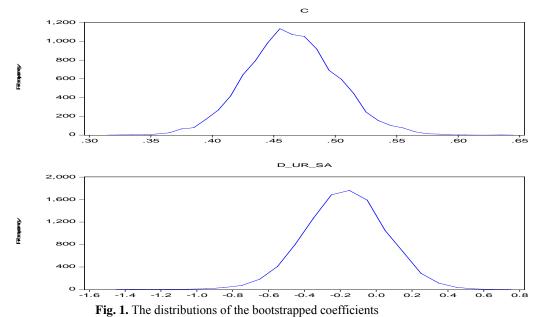
The data series were seasonally adjusted using Tramo/Seats method. The Tramo/Seats method has two phases. The first phase supposes the pre-adjustment of a time series. This consists in the adjustment for working days. In the Tramo/Seats method the adjustment for working days uses a regression model. The reason for pre-adjustment is the comparability of different observations in accordance to the working day structure. After the pre-adjustment of the data a linear time series model is fitted to it. The aim of this time series model is to show the interdependency apparent on the time axis of observations by means of a mathematical equation. The fitting of the model is comprised of the selection of the correct model and the coefficients' estimation.

The seasonally adjusted inflation rate is denoted by IR_SA. Moreover, the stationarity was checked using Augmented Dickey-Fuller test and the unemployment rate series was differentiated in order to have a stationary data set (the variable is denoted by D_UR_SA). The following regression model was valid, the coefficients estimated using ordinary least squares method-OLS- being bootstrapped with 10 000 replications of the residuals:

IR
$$SA = 0.464589048 - 0.1665851484*D$$
 UR SA (23)

According to Breusch-Godfrey test for lag 1, the errors are independent. The normality of error distribution is checked using Jarque-Bera test and we do not have reasons to accept the normality hypothesis. According to White test, the errors are homoscedastic. The results of these tests are presented in Appendix 1.

The predictions are made under the assumption that the unemployment rate keeps its value from the previous month.



The OLS forecasts (predictions based on the regression with parameters that were estimated using ordinary least squares-OLS- method) have a tendency to increase during the analyzed horizon. The limits of the intervals also increase. This growth can have important effects of the national level. In the context of convergence criteria, Romania will distance from the inflation target that was establish to achieve the inflation convergence. An expected increase in the unemployment rate will show an obvious prolongation of the economic crisis. From Table 1, the maximum value of inflation rate point forecast was anticipated for June 2014 (2.15%). The limits of the associated interval are higher than those of the previous prediction intervals. The

2.94

standard prediction interval is based on the OLS forecasts and the limits are computed considering the quintiles 2.5% and 97.5%. These quintiles imply certain critical values for z.

Inflation rate (%)	OLS forecasts	Standard prediction interval		
		2.5%	97.5%	
March 2014	1.82	1.03	2.61	
April 2014	1.95	1.16	2.74	
May 2014	2.03	1.24	2.82	

2.15

June 2014

Table 1. The OLS forecast and standard forecast intervals for average inflation rate (horizon: March 2014- June 2014)

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After the application of the two bootstrap forecast methods (percentile method and t-percentile one), a smaller range was observed for the percentile forecast intervals compared to standard intervals and t-percentile ones. This implies that this type of interval with smaller range has a lower degree of uncertainty. The inflation rate will grow over March 2014- June 2014, being between 1.03% and 2.61% in March 2014 and between 1.36% and 2.94% in June 2014. The consequences of this anticipated increase will put problems to the Romanian economy in the context of targeting regime and European convergence criterion. Therefore, the National Bank of Romania should continue the efforts to maintain a low inflation rate.

Table 2. The percentile and t-percentile prediction intervals for average inflation rate (horizon: March 2014- June 2014)

Inflation rate (%)	Percentile prediction interval		Percentile-t pre	ediction interval
	2.5%	97.5%	2.5%	97.5%
March 2014	2.62	3.33	2.75	3.80
April 2014	2.80	3.20	2.93	3.45
May 2014	2.85	3.34	2.95	3.47
June 2014	3.2	3.78	3.25	3.87

The inferior respectively superior limits of the bootstrapped intervals increase from one month to another. The results are in contradiction with the present policy in Romania that is oriented towards a continuous inflation decrease.

A moving average process was observed for the inflation rate (moving average model of order 1: MA(1)) and the historical errors method based on root mean square error was employed to predict the evolution of this indicator using forecast intervals. The MA process has the following form, with a backcast in the first month of 2004:

$$IR_SA = 0.4682778069 + [MA(1)=0.2511943256,BACKCAST=2004M01]$$

The MA model is used to make point predictions. The limits are computed considering the quintiles 2.5% and 97.5%.

Table 3. The point forecasts and the prediction intervals for average inflation rate based on historical errors method (horizon: March 2014- June 2014)

Inflation rate (%)	Point forecasts based on MA(1) process	Prediction interval	
		2.5%	97.5%
March 2014	0.93	0.86	1.34
April 2014	1.05	0.97	1.27
May 2014	1.33	1.19	1.42
June 2014	1.76	1.56	2.08

The predictions of inflation rate provided by this method seem to be more plausible, the point forecasts being below 2%. The range is quite low, showing that the forecast intervals are quite

optimistic. The previous intervals anticipated a quite large increase while these intervals are in accordance with the desired policy effects of National Bank.

Some dynamic and static forecasts are made for the analyzed period and some measures of quality forecasts were assessed.

The dynamic forecasts take into considerations the registered values but also the predictions made till that moment.

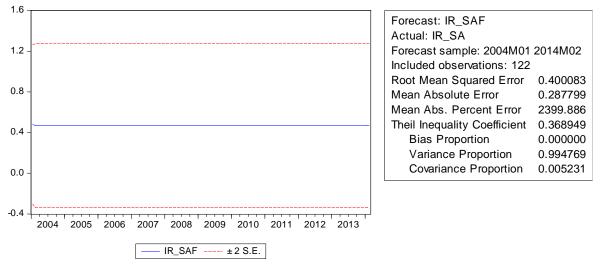


Fig. 2. Dynamic forecasts based on MA(1) model

The Theil's coefficient is greater than 0.25, the forecasts having a low degree of accuracy. The forecasts are unbiased and the proportion of variance is very high (almost 1). The root mean squared error (0.4) is larger than the mean absolute error (0.287).

The static forecasts are based only on the registered values, none of the predictions being taken into consideration.

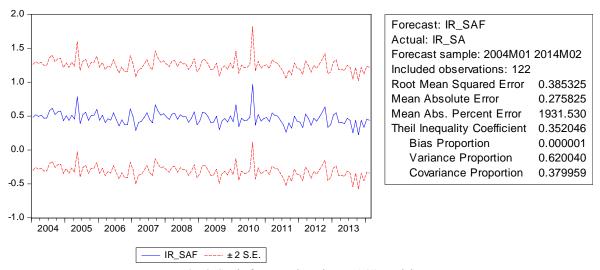


Fig. 3. Static forecasts based on MA(1) model

According to Theil's inequality coefficient with the value 0.35, the static forecasts are slowly superior in terms of accuracy compared to dynamic ones. Moreover, the root mean squared error

has a lower value for static forecasts. It is interesting that the predictions are not biased, the shocks in the economy being taken into account.

The variance proportion (0.62) shows how far the prediction variation is from the actual data variation. The covariance proportion (almost 0.38) shows the remaining unsystematic forecasting errors. The variance proportion is lower for static predictions, suggesting better forecasts than the dynamic ones.

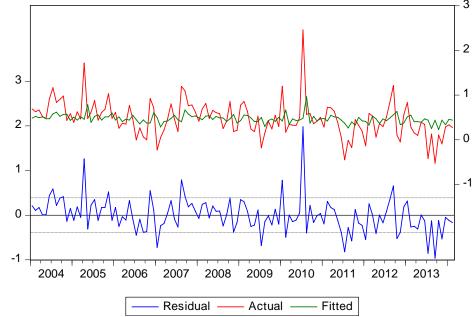


Fig. 4.The graphical representation of actual, fitted and residual values for the inflation rate modeled using MA(1) model

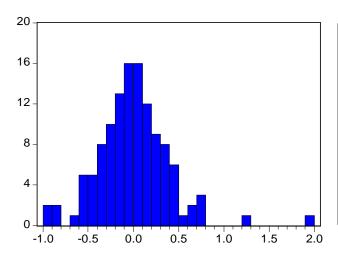
According to the previous graph, there are quite large differences between the fitted values of inflation based on moving average model and the registered value. The results are significant different from the National Bank anticipations and the inflation decrease policy. Even if the errors are homoscedastic and independent, only 7.26% of the variation in inflation is explained by the present and past errors of the process.

Conclusions

This paper is oriented towards the construction of forecast intervals for predicting the monthly inflation rate in Romania in order to improve the decisional process at macroeconomic level that uses the anticipation of inflation evolution. From the various forecasting methods, the bootstrap method was chosen in two variants (percentile and percentile-t forecast method). The results provided by these techniques are not too plausible, the range being slow and the boundaries being quite high. However, the historical errors method provided better results, even if the tendency is also of increase. After the application of the two bootstrap forecast methods (percentile method and t-percentile one), a smaller range was observed for the percentile forecast intervals compared to standard intervals and t-percentile ones. This implies that this type of interval with smaller range has a lower degree of uncertainty. The inflation rate will be subjected to growth over March 2014 - June 2014, being between 1.03% and 2.61% in March 2014 and between 1.36% and 2.94% in June 2014. The consequences of this anticipated increase will raise problems to the Romanian economy in the context of targeting regime and European convergence criterion. Therefore, the National Bank of Romania should continue the efforts to maintain a low inflation rate.

A future research could be directed to the proposal of new forecasting method for building prediction intervals. For example, some historical errors intervals could be built by taking into account the shocks in the Romanian economy. On the other hand, the assessment of forecast intervals could be made by using the suitable tests for evaluating the forecasts uncertainty. These chi-squared tests are based on the probability of success for the interval predictions.

APPENDIX 1



Series: Residuals Sample 2004:02 2014:02 Observations 121		
Mean	7.94E-17	
Median	-0.012813	
Maximum	1.980895	
Minimum	-0.992971	
Std. Dev.	0.401765	
Skewness	0.956930	
Kurtosis	7.403993	
Jarque-Bera	116.2508	
Probability	0.000000	

White Heteroscedasticity Test:

F-statistic	0.986697	Probability	0.375861
Obs*R-squared	1.990280	Probability	0.369672

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	4.96376 Probability	0.1233
Obs*R-squared	4.28672 Probability	0.1340

References

- 1. Alonso, M., Pena, D., Romo, J., Sieve bootstrap prediction intervals, *Proceedings in Computational Statistics 14th Symposium held in Utrecht*, The Netherlands, 2000, 181-186, 2000.
- 2. Efron, B., Tibhirani, R.J., An introduction to the Bootstrap, Chapman and Hall, New York.
- 3. Engel C. (2006). "Exchange-Rate Models". NBER Reporter Fall 2006, 17-34, 1993.
- 4. Gospodinov, N., "Median unbiased forecasts for highly persistent autoregressive processes". *Journal of Econometrics*, 111, 85-101, 2002.
- 5. Guan, W. 2003. From the help desk: bootstrapped standard errors. The Stata Journal (2003), 3, Number 1, pp. 71–80
- 6. M c C u 11 o u g h , B. D. (1994), Bootstrapping forecast intervals: An application to AR(p) models. J. Forecast., 13: 51–66. doi: 10.1002/for.3980130107
- 7. MacKinnon, J., Bootstrap inference in econometrics. Canad. J. Econom. v35. 615-645, 2002.
- 8. Miguel, J.A. and P. Olave (1999). Forecast intervals in ARCH models. Bootstrap versus parametric methods. *Applied Economics Letters*, 6, 323–327.
- 9. Reeves, R.R., Bootstrap intervals for ARCH models, *International Journal of Forecasting*, Vol. 21, pp. 237-248, 2005.