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Solving Problems of Investment Projects using the Programming with Whole Numbers or Using Lagrange's Multiplication

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Abstract

The purpose of the present work is to solve certain problems of economics which turn into programming problems with whole numbers or in problems of extremes with relations, for example different investment projects which in reality cover a lot of optimization problems. At the same time we offer mathematical models.

Key words: *investment project, programming with whole numbers, Lagrange's multiplication*

Problems: Selecting the Investment Projects

Nowadays, management techniques are becoming more and more familiar. Be using them, people find a way to solve the problems they encounter in an ever-changing environment.

Linear programming is a managerial technique used in solving those problems of allocating limited funds to different activities, in order to improve the final result.

Generally, the problems of fund implementation have the following characteristics:

- the implemented funds are limited;
- generally, the funds are used in the production goods and services;
- there is a different usage of the funds (solution or programme);
- the usage of the funds is limited by certain restrictions.

Linear programming is a technique used in solving a special type of problems in implementing funds- problems in which all the functions of the mathematical model are linear.

Here is an example of improving the investment using the linear programming:

Problem 1. A company wants to solve 'n' investment projects in a period of 'm' years.

Due to a limited budget just o part of them will be put into practice.

If the 'j' project brings to the company, in the case it is put into practice, a c_j profit, where j=1,…n, and if it brings every year an a_{ij} investment, where i+1,…m; and the value of the capital available for that year (i) is b_i , the problem resides in choosing the projects in such a way that the company should achieve maximum profit without exceeding the annual capital.

In order to solve the problem, the following variables are introduced:

$$x_j = \begin{cases} 1 \text{ if the "j' project is accepted} \\ 0 \text{ if the "j" project is refused} \end{cases}$$

Then, the problem resides in founding out xj, where j=1,…n when:

(1) $\mathrm{f} = \sum_{\mathrm{j}=1}^{\mathrm{n}} c_j x_j$ has the maximum value and the following restrictions occur:

(2) $\sum_{\mathrm{j}=1}^{\mathrm{n}} a_{ij} x_j \leq b_i ; i = 1,...m$

(3) $x_j \in \{0,1\}, j = 1,...n$

In practice $c_{j,} a_{ij}, b_i$ are considered to be whole positive numbers. This type of problem is a part of the general frame of linear programming and has the foregoing form with the difference that $x_j \in \mathrm{N}\ , x_j \geq 1$

Having the variable $y_1,...y_m$. then (2) can be written $\sum_{j=1}^{n} a_{ij} x_j + y_i = b_i$; $i = 1...m$; then we replace $y_1,...y_m$ with $x_{n+1...}, x_p$ where p = n+m and (2) can be written

$\sum_{j=1}^{p} a_{ij} x_j = b_i$ where $a_{ij} = 0$ for n+1≤ j≤p ; j≠n+i and $\mathrm{a}_{ij} = 1$ for j=n+i, $x_j = y_{n+i}$ and therefore the problem becomes, without restraining the generality:

(4) $f = \sum_{j=1}^{p} c_j x_j \ (c_{n+1} = ... = c_p = 0)$, take the maximum value

(5) $\sum_{j=1}^{p} a_{ij} x_j = b_i$

6) $\mathrm{x}_j \geq 0; c_j ; a_{ij}, x_j, b_i \in \mathrm{N}$ $\quad \bar{j} = \overline{1,p}$ *and* $\bar{i} = \overline{1,n}$

Problems {(4), (5),(6)}=type I is called linear programming with whole numbers.

If (7) when x≥0, problems{(4},(5),(7)}=type II are called linear programming (in the general acceptance of the term).

The R. E. Gomory Algorithm

Step 1: Using the Simplex algorithm applied to (5) we consider:

$\mathrm{T} \subset \{1, 2,...\mathrm{p}\}$ -indices of basic variables

$\mathrm{J} \subset \{1, 2\ ...\mathrm{p}\} - \mathrm{I}$ - indices of non-basic variables . From (5) in A

X=B ; $A = (a_{ij})_{\substack{i=1,m \\ j+1,p}}$; $X = \begin{pmatrix} x_1 \\ x_2 \\ ... \\ x_p \end{pmatrix}$; $B = \begin{pmatrix} b_1 \\ b_2 \\ ... \\ b_p \end{pmatrix}$ we arrive at (8) $x_i + \sum_{j \in J} \overline{a_{ij}} x_j = \overline{b_i}$

meaning that basic(principle) variables are presented: $(x_i), i \in \{1...r\} = I$ according to secondary variables $(x_j), j \in J$ (Simplex algorithm).

The $x_i^o = \overline{b_i}; i \in I$ *and* $x_j^0 = 0; j \in$ J values are the perfect solution for the problem (II)

Step 2: If $\overline{b_i} \in \mathbf{N}$, i∈ 1,…m, we come up with a solution for the problem (type I). In this case we come to the following restrictions:

(9) $\sum_{j \in J} \{\overline{a_{rj}}\} x_j \geq \{\overline{b_r}\}$ where $\{\overline{a_{rj}}\}; \{\overline{b_r}\}$ fractional parts of $\overline{a_{rj}}$ and $\overline{b_r}$. The demonstration of this restriction in solving a wide variety of problems is found at (4) point (.See bibliography.)

Introducing a new variable x_{p+1} in (8) we deduce a new equality, meaning, a new equation:

$$-\sum_{j \in J} \{\overline{a_{rj}}\} x_j + x_{n+1} = \{b_r\}$$

Adding the equation to the system used in (8);to the Simplex algorithm from the preceding steep and results (11) : $x_i^0 = \overline{b_i}\ i \in I;\ x_j^0 = 0, j \in J; x_{p+1} = \{\overline{b_r}\}$ which is a new solution of the general programming problem.

Step 3: When all the solutions from (11) become whole numbers it means that we have determined solutions in whole numbers for the linear programming problem.

Problem 2. We want to find $x_1, ...x_n$ so that the function $f = c_1x_1 + c_2x_2 + .. + c_nx_n$ have an extreme value. In problem

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + .. + a_{1n}x_n = b_1 \\ .... \\ a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = b_m \end{cases} \quad \text{cu } x_j \geq 0 \ (\forall)\, j = 1,...n$$

We could see the problem at type(I)as one of extremes with relations; looking for the initial solution to the general.

Step 1: We introduce the function:

$$L(x_1, x_2, ...x_n, \lambda_1, \lambda_2, ...\lambda_m) = c_1x_1 + c_2x_2 + .. + c_nx_n + \lambda_1(a_{i1}x_1 + a_{i2}x_2 + .. + a_{in}x_n - b_1) +$$
$$\lambda_2(a_1x_1 + a_2x_2 + ...a_{2n}x_n - b_2) + ... + \lambda_m(a_{m1}x_1 + a_{m2}x_2 + ... + amnx_n - b_m)$$

We determine the critical points of this function using the system $L'_{x_1} = 0$,

$$L'_{x_n} = 0, L'_{\lambda_1} = 0 ... L'_{\lambda_m} = 0 \text{ which becomes: } \begin{cases} \lambda_1 a_{11} + \lambda_1 a_{21} + ... + \lambda_m a_{m1} = -c_1 \\ \lambda_1 a_{12} + \lambda_2 a_{22} + ... + \lambda_m a_{m2} = -c_2 \\ .... \\ \lambda_1 a_{1n} + \lambda_2 a_{2n} + ... + \lambda_m a_{mn} = -c_n \\ a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1 \\ .... \\ a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = b_m \end{cases}$$

Which mean we have solved the m+n system of equation of $\lambda_1, \lambda_{2...}\lambda_m, x_{1,}, x_2...x_n$. If the system doesn't have solutions in general (positive), then it won't have solution with whole numbers either. So we proceed to the next step.

Step 2: Be it: $\mathbf{x_1^0...x_n^0}; \lambda_1^0, ...\lambda_m^0$ a solution of the previous system introducing the function

$$L(x_1,..x_n) = c_1x_1 + ... + c_nx_n + \lambda_1(a_{11}x_1 + ... + a_{1n}x_n - b_1) + ... + \lambda_m(a_{m1}x_1 + ... + a_{mn}x_n - b_m)$$

considered variables just in $x_1 ... x_n$ it is obvious that $x_1^0 ... x_n^0$ is the critical point of this function. Writing the difference of L:

$$dL = L'_{x_1} * h_1 + ... + L'_{\lambda_n} * h_n = (c_1 + \lambda_1 a_{11}) h_1 + ... + (c_n + \lambda_m a_{mn}) h_n$$

and differencing the relations:

$$\left.\begin{aligned} g_1 = a_{11}x_1 + ... + a_{1n}x_n - b_1 = 0 \Rightarrow dg_1 = a_{11}h_1 + ... + a_{1n}h_n = 0 \\ g_m = a_{m1}x_1 + ... + a_{mn}x_n - b_m = 0 \Rightarrow dg_m = a_{m1}h_1 + ... + a_{mn}h_n = 0 \end{aligned}\right\} (*)$$

We will evaluate dL in conditions (*) and if is positively or negatively defined in $x_1^0, ...x_n^0$ this ($x_1^0, ...x_n^0$) will be extreme, minimum or maximum.

Step 3: Between the positive solutions $x_1^0, ... x_n^0$ (if it is the case) we are looking for the adequate one by verifying it according to the purpose.

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Rezolvarea unor probleme de proiectare a investițiilor cu ajutorul programării în numere întregi sau cu ajutorul multiplicatorilor lui Lagrange

Rezumat

Lucrarea îşi propune să rezolve anumite probleme economice ce se transpun în probleme de programare în numere întregi sau pur şi simplu în probleme de extreme cu legături, cum ar fi diferite proiecte de investiții care în practică acoperă o mulțime de aspecte ale optimizării. Pentru una din acestea am oferit modele matematice.